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NAVAL POSTGRADUATE SCHOOL

Monterey, California

NAVAL APPLICATIONS:
TEN ALGORITHMS FOR THE HEWLETT-PACKARD
HP-67 AND HP-97 CALCULATORS

edited by

R. H. Shudde

February 1979

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NAVAL POSTGRADUATE SCHOOL
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Sonar

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Sonar	Crew Management	Normal Mode	Programmable Calculator
Active Sonar	Target Motion Analysis	TMA	
Sonar Acquisition	Golden Section Search	NGFS	HP-67 Calculator
P-3 Aircraft	Transmission Loss	Simulation	Bearings-Only
Mission Planning	Gunfire support	Calculator	Tracking

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

Ten algorithms pertaining to underwater acoustics target motion analysis, P-3 mission planning, flight crew management, and naval gunfire support conversion are presented along with programs for Hewlett-Packard HP-67 and HP-97 programmable calculators.

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TABLE OF CONTENTS

ABSTRACT

Ten algorithms pertaining to underwater acoustics, target motion analysis, P-3 mission planning, flight crew management, and naval gunfire support conversions are presented along with programs for Hewlett-Packard HP-67 and HP-97 programmable calculators.

I. INTRODUCTION

This report contains a collection of programs which were submitted by officers in partial fulfillment of the requirements of the course Tactical Design and Analysis (OA 4658) conducted at the Naval Postgraduate School during the period of October through December 1978.

All programs were listed using an HP-97 with HP-97 key codes. The corresponding HP-67 key codes may be found on pages 324 through 331 of the "HP-67 Owner's Handbook and Programming Guide."

II. ACTIVE SONAR ACQUISITION by Mr. R. F. Fish and LT M. H. Trent

A. Problem Statement

A sonar at a depth (SD) has the possiblity of detecting a target at a depth TD at a slant range r'. Detection can only occur if the target lies within the beam pattern and the signal excess is at least equal to the detection threshold. Whether or not the system is noise or reverberation limited depends on the geometry and doppler frequency shift. The problem is to determine the acquisition range of the sonar with various geometries and acoustic parameters.

B. Operational Analysis

The analysis uses a 0 dB detection threshold because of the limited number of storage registers (26) and program steps (224) in the calculator.

In using the program it should be noted that calculations do not include the effect of shadow zones. Acquisition ranges computed must be considered with this in mind. In addition, once the target and surface signals are outside the beam pattern ($\pm$ 3db) they are assumed to abruptly disappear respectively. The analysis also assumes the water is deep with no bottom effects.

Considering the above caveats the source and target can be placed as desired in the medium and the appropriate sonar equation parameters will be computed. In doing so, several tests will be made to determine which equations will be used.

The program will terminate before acquisition if one of the following occurs:

Slant range

$r' < 0$,

Angle-to-target

$\theta_1 > \phi/2 + \gamma$, or $\theta_1 > \phi/2 - \gamma$ (if target is below source),

Angle-to-surface at r'

$\theta_2 > \phi/2 + \gamma$,

where

γ = pitch angle range.

These terminations and signal excess $SE \geq 0$ will finish with a "1" printed as an output at the end of the calculations.

After calculation of the surface reverberation level, RL_s, there is a program stop where the appropriate correction can be input for off-axis transmission and reception. The same event occurs when SE is calculated so that the sonar equation can be corrected.

The output listing is as follows:

Doppler	No Doppler
r'	r'
θ_1	θ_1
θ_2	θ_2
TL	TL
NL_s	RL_s displays but not printed
SE	Total of $RL_s + RL_v + NL_s$ combined and SE

C. Computational Algorithm

1. Input

   --Sound speed, c (m/sec)

   --Listening time between transmit pulses, t (sec)

   --Source depth, SD (m)

   --Target depth, TD (m)

   --Horizontal (and vertical) half-beam width, $\phi/2$ (degrees)

   --Sonar pitch angle, γ (degrees)

   --Mixed layer depth, D (meters)

   --Absorption coefficient, α (dB/meter)

   --Frequency, f (kHz)

   --Sea state, S.S.

   --Column scattering coefficient, S_c

   --Constant, 10

   --Sonar self-noise level, NL_s (dB)

   --Target strength, TS (dB)

   --Range decrement (meters)

   --[Pulse length, τ (sec) $\times$ ϕ (radians)] $\div$ 2

   --Surface scattering coefficient, S_s (dB)

   --Sonar source level, SL (dB)

2. For doppler set Flag 0; for no doppler clear Flag 0.

3. Output

   r', θ_1, θ_2, TL, RL_s, RL_v. total level, SE. For doppler, RL_s, RL_v, and total level are included in NL_s.

D. HP-67/97 Calculator Program

1. User Instructions

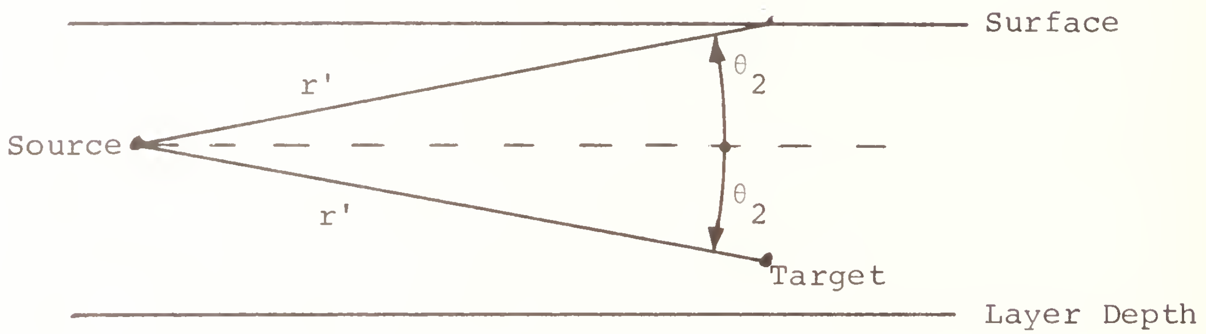
Step	Instruction	Input	Key(s)	Output
1.	Enter program card			
2.	Enter data in primary stage:			
a.	Sound speed (m/sec)	c	STO 0	
b.	Listening time (sec)	t	STO 1	
c.	Source depth (m)	SD	STO 3	
d.	Target depth (m)	TD	STO 4	
e.	Half-beam width (deg)	$\phi/2$	STO 7	
f.	Sonar pitch angle (deg)	γ	STO 8	
g.	Mixed layer depth (m)	D	STO A	
h.	Absorption coefficient (dB/m)	α	STO B	
i.	Frequency (kH_z)	f	STO C	
j.	Sea state	S.S.	STO D	
k.	Source level (db)	SL	h STI	
3.	Enter data in secondary storage:*		P ⇄ S	
a.	Column scattering coefficient	S_c	STO 0	
b.	Constant	10	STO 2	
c.	Sonar self-noise level	NL_s	STO 3	
d.	Target strength	TS	STO 4	
e.	Range decrement (m)	r.d.	STO 5	
f.	$[\tau(\text{sec}) \times \phi(\text{radians})]/2$	$\tau\phi/2$	STO 6	
g.	Surface scattering coefficient	S_s	STO 8	
4.	Primary/Secondary exchange*		P ⇄ S	
5.	Doppler or No doppler		SF 0 CF 0	doppler no doppler
6.	Start computations	--	A	See Step 7
7.	Printed output:			
a.	Slant range			r'
b.	Angle-to-target			θ_1
c.	Angle-to-surface at range r'			θ_2
d.	Transmission Loss			TL
e.	If Flag 2 is set, self-noise is printed.			NL_s

Step	Instruction	Input	Key(s)	Output
8.	If Flag 2 is set, go to Step 9. Otherwise:			
a.	Display surface reverberation level			RL_s
b.	Enter two-way beam pattern correction in db (0 if no correction)	Correction	R/S	Total
	Print total corrected level.			RL_s
9.	Stop and display SE. To change range decrement, execute Step a. Otherwise go to Step b.			SE
a.	Key in new range decrement	r.d.	STO 5 R ↓	SE
b.	Enter two-way beam pattern correction in db (0 if no correction)	Correction	R/S	Corrected
	Display corrected SE			SE
10a.	If SE $<$ 0, execution continues from Step 7.			
b.	If SE $\geq$ 0, termination occurs			1.00

*The primary and secondary registers must be exchanged before (Step 3) and after (Step 4) entering data into the secondary storage registers.

2. Sample Problems

a. Sonar and target are in the mixed layer in a "doppler" situation, so that acquisition is noise limited by NL_s (Figure 1).



Source depth = 30 meters
Target depth = 60 meters
Mixed Layer depth = 75 meters

FIGURE 1. Geometry of Sample Problem 1.

Input.

R0: 1500 m/sec

R1: 6.7 sec

R3: 30m

R4: 60m

R7: 10°

R8: 5°

RA: 75m

RB: .00328 dB/m

RC: 25 kHz

RD: 2

RI: 227 dB re 1μPa @ 1 m

S0: -50 dB

S2: 10

S3: 65 dB re 1μPa

S4: 10 dB

S5: 500 m (initial range decrement)

S6: .17°sec

S8: -30 dB

Set Flag 0. No corrections are made to SE.

Output

The results are shown on the Sample Problem output.

At acquisition

$r' = 3650$ meters

$\theta_1 = .47$

$\theta_2 = .47$

TL = 85.9 dB (layer)

$NL_s = 65$ dB re 1μPa

SE = .21 dB

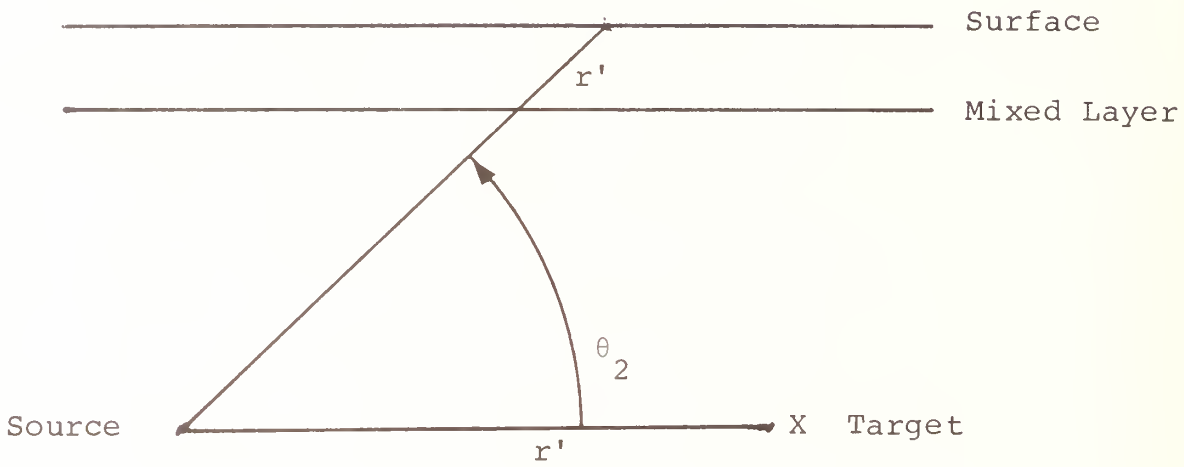
1500.00 STO0 6.70 STO1 30.00 STO2 50.00 STO4 10.00 STO7 5.00 STO8 75.00 STO9 .00326 STOE 35.00 STOC 2.00 STOD 227.00 STOI P⇄S	Primary Storage Registers
-50.00 STO0 10.00 STO2 65.00 STO3 10.00 STO4 500.00 STO5 .17 STO6 -30.00 STO8 P⇄S	Secondary Storage Registers

SAMPLE PROBLEM 1. Input Data

	SF0	Set for doppler
	GSBa	Start
5025.00	***	r'
0.34	***	θ_1
0.34	***	θ_2
93.76	***	TL
65.00	***	RL_S
0.00	R/S	Correction to SE
-15.52	***	Corrected SE
4525.00	***	
0.36	***	
0.38	***	
90.95	***	
65.00	***	
0.00	R/S	
-9.90	***	
4025.00	***	
0.43	***	
0.43	***	
88.09	***	
65.00	***	
300.00	STO5	Change range decrement
	R↓	to 300 meters
0.00	R/S	
-4.17	***	
3725.00	***	
0.46	***	
0.46	***	
86.34	***	
65.00	***	
50.00	STO5	Change range decrement
	R↓	
0.00	R/S	
-0.67	***	
3675.00	***	
0.47	***	
0.47	***	
86.04	***	
65.00	***	
25.00	STO5	Change range decrement
	R↓	
0.00	R/S	
-0.09	***	
3650.00	***	r'
0.47	***	θ_1
0.47	***	θ_2
85.90	***	TL (layer)
65.00	***	NL_S
0.00	R/S	
0.21	***	SE = 0.21
1.00		Terminate

SAMPLE PROBLEM 2. Output

b. Sample Problems 2 and 3. The sonar and target configuration are shown in Figure 2.



Source depth = 600 meters
Target depth = 600 meters
Mixed Layer depth = 75 meters

FIGURE 2. Geometry of Sample Problems 2 and 3.

Input for Problems 2 and 3:

Same as for Problem 1 except

R3: 600 meters

R4: 600 meters

R8: 0°

Flag 0: set for problem 2; clear for problem 3

0 entered as correction factors to RL_s and SE.

Output for Problem 2:

$r' = 4125$ meters

$\theta_1 = 0°$

$\theta_2 = 8.36°$

SE = .32 dB

Sonar acquired the target at about 4125 meters.

Output for Problem 3:

$r' = 3025$ meters

$\theta_1 = 0°$

$\theta_2 = 11.44°$

SE = 0.85 dB

Sonar acquired at 3025 meters.

	SF0	
	GSBA	
5025.00	***	
0.00	***	
6.86	***	
90.50	***	
65.00	***	
0.00	R/S	
-9.01	***	
4525.00	***	
0.00	***	
7.62	***	
87.95	***	
65.00	***	
100.00	STOE	Change range decrement to
	R↓	100 meters
0.00	R/S	
-3.91	***	
4425.00	***	
0.00	***	
7.79	***	
87.43	***	
65.00	***	
0.00	R/S	
-2.86	***	
4325.00	***	
0.00	***	
7.97	***	
86.91	***	
65.00	***	
0.00	R/S	
-1.81	***	
4225.00	***	
0.00	***	
8.16	***	
86.37	***	
65.00	***	
0.00	R/S	
-0.75	***	
4125.00	***	r'
0.00	***	θ_1
8.36	***	θ_2
85.84	***	TL
65.00	***	NL_s
0.00	R/S	
0.32	***	SE
1.00		Terminate

SAMPLE PROBLEM 2. Output

	CF3	No doppler
	GSBA	Start
5025.00	***	r'
0.00	***	θ_1
6.66	***	θ_2
90.50	***	TL
77.07	***	RLS
0.00	R/S	No correction to RL_s
77.37	***	Total RL_s
0.00	R/S	No correction to SE
-21.38	***	Corrected SE
4525.00	***	
0.00	***	
7.62	***	
87.95	***	
81.71	***	
0.00	R/S	
81.65	***	
0.00	R/S	
-20.76	***	
4025.00	***	
0.00	***	
8.57	***	
85.30	***	
86.52	***	
0.00	R/S	
86.59	***	
0.00	R/S	
-20.19	***	
3525.00	***	
0.00	***	
9.80	***	
82.51	***	
91.53	***	
0.00	R/S	
91.58	***	
0.00	R/S	
-19.59	***	
3025.00	***	r'
0.00	***	θ_1
11.44	***	θ_2
79.54	***	
77.08	***	
0.85	***	
0.00	R/S	
0.85	***	SE
1.00		Termination

SAMPLE PROBLEM 3. Output

3. Program Storage Allocations and Program Listings

Registers

R0:	c (m/sec)	S0:	S_c (dB)
R1:	t (sec) and	S1:	RL_v (dB)
	SL - 2TL + 10 log r'	S2:	10
R2:	R_{max} and r'	S3:	NL ($dB_{rel\,\mu Pa}$)
R3:	SD (m)	S4:	TS (dB)
R4:	TD (m)	S5:	Range decrement (m)
			(500 m default)
R5:	θ_1 (deg)		
R6:	θ_2 (deg)	S6:	τ (sec) $\times$ $\phi/2$ (AD)
R7:	$\phi/2$ (deg)	S7:	10 log($\phi\tau c/2$)
R8:	γ (deg)	S8:	S_s (dB)
R9:	$\phi/2 + \gamma$ (deg)	S9:	RL_s (dB)
RA:	D (meters)		
RB:	α (dB/meter)		
RC:	f (kHz)		
RD:	S.S.		
RE:	TL (dB)		
RI:	SL ($dB_{rel\,\mu Pa}$ @ 1m)		
	or consistent with NL		

Initial Flag Status and Use:

0: ON for doppler,
OFF for no doppler

1, 2, 3: OFF, unused

User control keys:

A: Start program	a:
B:	b:
C:	c:
D:	d:
E:	e:

001	*LBLA	21 11	038	X>Y?	16-34	075	X>Y?	16-34
002	RCL0	36 00	039	GTO0	22 00	076	GTO6	22 06
003	RCL1	36 01	040	*LBL1	21 01	077	LOG	16 32
004	×	-35	041	RCL7	36 07	078	1	01
005	2	02	042	RCL8	36 08	079	0	00
006	÷	-24	043	-	-45	080	×	-35
007	STO2	35 02	044	RCL5	36 05	081	RCL2	36 02
008	*LBLB	21 12	045	X>Y?	16-34	082	LOG	16 32
009	RCL2	36 02	046	GTO0	22 00	083	1	01
010	X<0?	16-45	047	RCLA	36 11	084	0	00
011	GTO0	22 00	048	RCL3	36 03	085	×	-35
012	PRTX	-14	049	X>Y?	16-34	086	+	-55
013	1/X	52	050	GTO6	22 06	087	RCLE	36 15
014	RCL3	36 03	051	X⇄Y	-41	088	RCL2	36 02
015	RCL4	36 04	052	RCL4	36 04	089	×	-35
016	-	-45	053	X>Y?	16-34	090	+	-55
017	ABS	16 31	054	GTO6	22 06	091	RCLC	36 13
018	×	-35	055	RCL3	36 03	092	√X	54
019	SIN⁻¹	16 41	056	RCL4	36 04	093	RCLD	36 14
020	STO5	35 05	057	X≤Y?	16-35	094	×	-35
021	PRTX	-14	058	X⇄Y	-41	095	1	01
022	RCL3	36 03	059	CHS	-22	096	.	-62
023	RCL2	36 02	060	RCLA	36 11	097	0	00
024	÷	-24	061	+	-55	098	4	04
025	SIN⁻¹	16 41	062	X=0?	16-43	099	×	-35
026	STO6	35 06	063	GTO6	22 06	100	RCL2	36 02
027	PRTX	-14	064	1/X	52	101	×	-35
028	RCL7	36 07	065	RCLA	36 11	102	RCLA	36 11
029	RCL8	36 08	066	X²	53	103	√X	54
030	+	-55	067	×	-35	104	8	08
031	STO9	35 09	068	√X	54	105	4	04
032	RCL4	36 04	069	1	01	106	0	00
033	RCL3	36 03	070	0	00	107	×	-35
034	X≤Y?	16-35	071	5	05	108	÷	-24
035	GTO1	22 01	072	×	-35	109	+	-55
036	RCL9	36 09	073	RCL2	36 02	110	STOE	35 15
037	RCL5	36 05	074	X⇄Y	-41	111	GTOC	22 13

```
112  *LBL6     21 06
113   RCL2     36 02
114    LOG     16 32
115      2        02
116      0        00
117      x       -35
118   RCLB     36 12
119   RCL2     36 02
120      x       -35
121      +       -55
122   STOE     35 15
123  *LBLC     21 13
124   FRTX       -14
125   RCLI     36 46
126   RCLE     36 15
127      2        02
128      x       -35
129      -       -45
130   RCL2     36 02
131    LOG     16 32
132      1        01
133      0        00
134      x       -35
135      +       -55
136   STO1     35 01
137   RCL0     36 00
138    P≠S     16-51
139   RCL6     36 06
140      x       -35
141    LOG     16 32
142      1        01
143      0        00
144   STO2     35 02
145      x       -35
146   STO7     35 07
147    P≠S     16-51
148    F0?  16 23 00
```

```
149   GTOE     22 15
150   RCL9     36 09
151   RCL6     36 06
152   X>Y?     16-34
153   GTOD     22 14
154   RCL1     36 01
155    P≠S     16-51
156   RCL7     36 07
157      +       -55
158   RCL8     36 08
159      +       -55
160   STO9     35 09
161    R/S        51
162      -       -45
163   GTOa  22 16 11
164  *LBLD     21 14
165    P≠S     16-51
166      0        00
167   STO9     35 09
168  *LBLa  21 16 11
169   RCL0     36 00
170   RCL7     36 07
171      +       -55
172    P≠S     16-51
173   RCL1     36 01
174      +       -55
175    P≠S     16-51
176   STO1     35 01
177   RCL2     36 02
178   RCL9     36 09
179   RCL2     36 02
180      ÷       -24
181     Y^X       31
182   RCL2     36 02
183   RCL1     36 01
184   RCL2     36 02
185      ÷       -24
186     Y^X       31
```

```
187      +       -55
188    P≠S     16-51
189  *LBLE     21 15
190    P≠S     16-51
191   RCL3     36 03
192   RCL2     36 02
193      ÷       -24
194   RCL2     36 02
195    X≠Y       -41
196     Y^X       31
197      +       -55
198    LOG     16 32
199   RCL2     36 02
200      x       -35
201   FRTX       -14
202    CHS       -22
203   RCLI     36 46
204      +       -55
205   RCLE     36 15
206      2        02
207      x       -35
208      -       -45
209   RCL4     36 04
210      +       -55
211    R/S        51
212      -       -45
213   FRTX       -14
214   X<0?     16-45
215   GTO9     22 09
216  *LBL0     21 00
217      1        01
218   FRTX       -14
219    RTN        24
220  *LBL9     21 09
221   RCL5     36 05
222    P≠S     16-51
223   ST-2  35-45 02
224   GTOB     22 12
```

E. Computational Analysis

The active sonar equation is

$$SL - 2TL + TS - \begin{matrix}(NL - DI)\\ NL_s \\ RL\end{matrix} \geq DT \ ,$$

where

SL = source level for the sonar ($dB_{rel\mu Pa}$ @ 1m),

TL = transmission loss (dB),

(NL-DI) = ambient noise term which is neglected due to a higher NL_s or RL,

RL = reverberation level (dB)

DT = system detection threshold (0dB assumed),

TS = target strength (dB),

and NL_s = Self-noise ($dB_{rel\mu Pa}$).

The only terms not known in the equation, TL and RL_s, are calculated at various ranges (decrements) until the signal excess (SE) $\geq$ DT(0).

The TL is calculated for two conditions:

a. When both source and target are in the layer (Reference 1)

$$TL = 10 \log r_t + 10 \log r' + \alpha r' + \frac{br'}{rs} \ ,$$

where

$r_t = 105\sqrt{\frac{D^2}{D - z_s}}$ is the transition range (meters),

α = absorption (dB/meters),

$b = 1.04 \times SS \times \sqrt{f}$ bounce factor (dB/bounce) valid between (3-25 kHz)(3-14 dB/bounce),

$r_s = 840 \sqrt{D}$,

z_s = larger of source or receiver depths (meters),

and D = layer thickness (meters).

b. When both source and target are not in the layer the TL is

$$TL = 20 \log r' + \alpha r' \quad (\text{for } r' < r_t \text{ also when in layer}).$$

After using the proper TL formula it must be decided whether or not there is sufficient doppler to be able to disregard reverberation (i.e., RL_s and RL_v). If there is enough doppler then the NL_s term dominates and the sonar equation can be solved. If there is no doppler the appropriate reverberation must be considered and combined with NL_s. Then the sonar equation can be solved. By successive decrements of r', there *may* be a point where $SE \geq DT$ and thus detection has occurred.

The reverberation equations are (Reference 2),

$$RL_s = SL - 2TL + 10 \log r' + S_s + 10 \log \left(\frac{\phi C \tau}{2}\right) ,$$

where

S_s = surface scattering parameter (dB) for the particular conditions (wind speed, grazing angle),

ϕ = sonar horizontal beam width (radians),

c = wave propagation time

and τ = transmit pulse width (seconds);

and

$$RL_v = SL - 2TL + 10 \log r' + S_c + 10 \log \left(\frac{\phi C \tau}{2}\right) ,$$

where

S_c = column scattering coefficient (dB) for the particular environmental conditions.

At each range decrement θ_1 and θ_2 are calculated to determine if they are inside the beam pattern. If θ_1 is not, acquisition cannot occur. If θ_2 is not, then RL_s is not important. The formulae for these quantities are (Figure 3):

$$\theta_1 = \sin^{-1} \left|\frac{SD - TD}{r'}\right| \quad \text{and} \quad \theta_2 = \sin^{-1} \frac{SD}{r'} ,$$

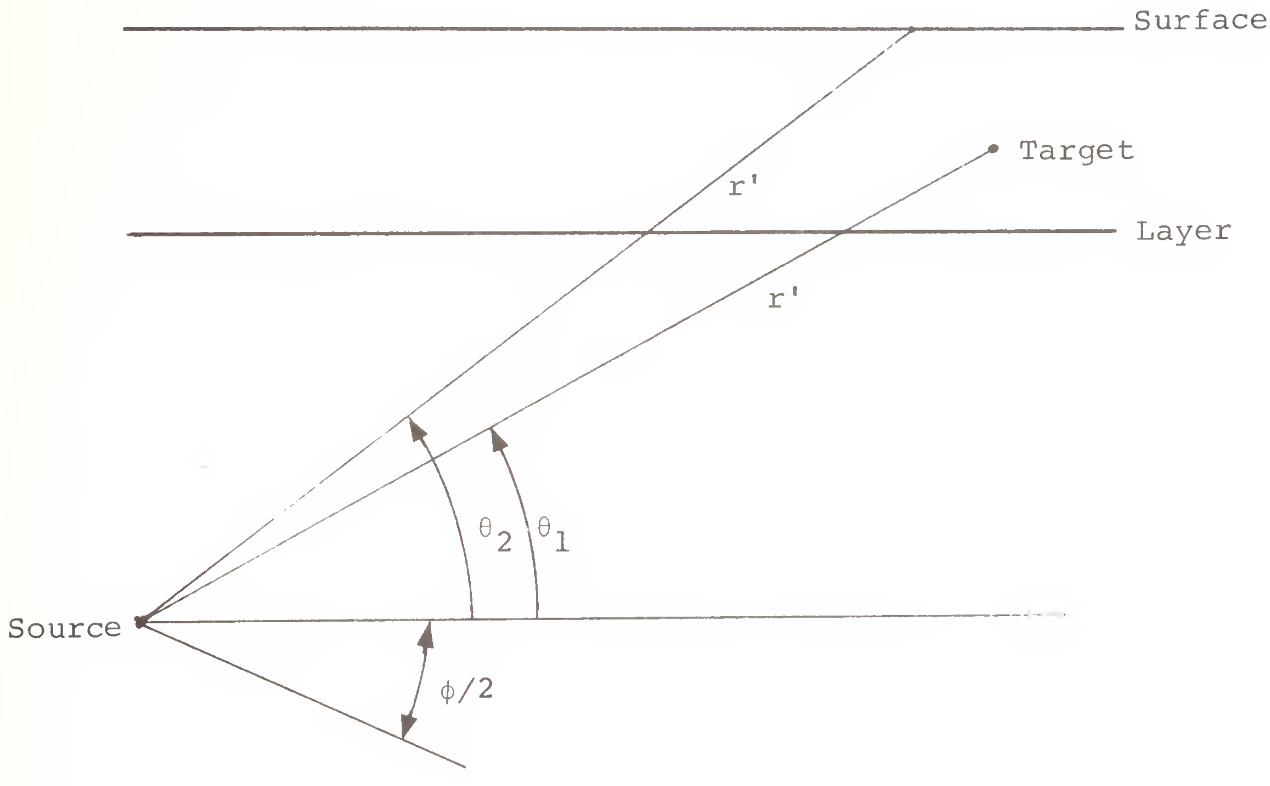
where

SD = source depth,

and TD = target depth.

In addition, depending on the values of these angles, corrections can be made to the RL_s and sonar equation to compensate for off-axis (beam pattern) transmission and reception.

To get an initial value of $r' = R_{max}$ the equation $R_{max} = ct/2$ is used where t = the sonar "listening" time or time between successive pulse transmission.



ϕ = sonar beam pattern (3dB points)

θ_1 = angle to target at range r'

θ_2 = angle to surface at rnage r'

r' = slant range

D = layer depth

FIGURE 3. Source and Target Geometry

F. References

1. A. B. Coppens and J. V. Sanders, "An Introduction to the Sonar Equations with Applications," Technical Report NPS-61Sd76071, July 1976, Naval Postgraduate School, Monterey, CA 93940.

2. R. J. Urick, Principles of Underwater Sound, 2nd Edition McGraw-Hill Book Co., 1975.

III. THREE ENGINE AVAILABLE RANGE REMAINING: P-3(B) AIRCRAFT CONFIGURATION "B" by LT R. J. Knight

A. Problem Statement

Aircraft total fuel remaining, outside air temperature, and aircraft altitude data are available. Determine the aircraft's available range remaining.

This program allows a pilot or copilot to rapidly and efficiently provide a quick estimate of available range remaining in an emergency situation (three engine flight).

B. Operational Analysis

The aircraft's available range remaining can be extracted from the table listed on pages 12-189 of the P-3(B) aircraft NATOPS manual.

C. Computational Algorithm

1. Input fuel remaining (pounds).
2. Input outside temperature (°C).
3. Input altitude (feet)
4. Calculate the three-engine available range remaining.

D. HP-67/97 Calculator Program

1. User Instructions

Step	Instruction	Input	Key(s)	Output
1.	Load program magnetic card side #1 and side #2			0.000000000
2.	Input fuel remaining,* press enter	pounds	ENT	pounds
3.	Input outside air temperature for specific altitude press enter	°C	ENT	°C
4.	Input altitude	feet		feet
5.	Press A to calculate the three engine available range remaining		A	range in NM

---

*Note: Fuel remaining must be entered as #10,000, #20,000, #30,000, #40,000 or #50,000. "Error" will display otherwise.

2. Sample Problem

Calculate the three-engine available range remaining if the remaining fuel is 10,000 pounds, the outside temperature is 11°C, and the altitude is 2000 feet. (ANSWER: 295 n.mi.)

```
10000. ENT↑
   11. ENT↑
 2000. GSBA
  295.  ***
```

3. Program Storage Allocation, Permanent Data, and Program Listing

Registers:

R0: a_0	S0:	A: Fuel
R1: b_0	S1:	B: Altitude
R2: a_1	S2:	C: Temperature
R3: b_1	S3:	D: Temperature deviation
R4: a_2	S4:	E: Uncorrected range.
R5: b_2	S5:	
R6: a_3	S6:	
R7: b_3	S7:	
R8: a_4	S8:	
R9: b_4	S9:	

Flags: OFF, unused.

User Control Keys:

A: Compute	a.
B:	b:
C:	c:
D:	d:
E:	e:

Permanent Data

The following permanent data are stored in the primary storage registers R0 through R9.

-34302.26904	0
122.9065370	1
-35846.62839	2
43.18615709	3
-36857.62093	4
27.00641670	5
-38068.60561	6
20.16918143	7
-39482.60271	8
16.50293201	9

Program Listing

*LBLA	21 11	START
STOB	35 12	STO ALTITUDE
R↓	-31	
STOC	35 13	STO TEMP
R↓	-31	
STOA	35 11	STO FUEL
EEX	-23	
4	04	TEST FUEL
÷	-24	= 10,000 lbs?
1	01	
-	-45	
X=0?	16-43	
GTO1	22 01	
1	01	TEST FUEL
-	-45	= 20,000 lbs?
X=0?	16-43	
GTO2	22 02	
1	01	TEST FUEL
-	-45	= 30,000 lbs?
X=0?	16-43	
GTO3	22 03	
1	01	TEST FUEL
-	-45	= 40,000 lbs
X=0?	16-43	
GTO4	22 04	
1	01	TEST FUEL
-	-45	= 50,000 lbs
X=0?	16-43	
GTO5	22 05	
X⇄Y	-41	TEST FUEL
0	00	> 50,000 lbs
÷	-24	
*LBL1	21 01	
RCLB	36 12	COMPUTE RANGE
RCL0	36 00	W/O TEMP CORRECTION
-	-45	F = 10,000 lbs
RCL1	36 01	
÷	-24	
STOE	35 15	
GTOe	22 16 15	
*LBL2	21 02	COMPUTE RANGE
RCLB	36 12	W/O TEMP CORRECTION
RCL2	36 02	F = 20,000 lbs
-	-45	
RCL3	36 03	
÷	-24	
STOE	35 15	
GTOe	22 16 15	
*LBL3	21 03	COMPUTE RANGE
RCLB	36 12	W/O TEMP CORRECTION
RCL4	36 04	F = 30,000 lbs
-	-45	
RCL5	36 05	
÷	-24	
STOE	35 15	
GTOe	22 16 15	
*LBL4	21 04	
RCL4	36 04	COMPUTE RANGE
RCLB	36 12	W/O TEMP CORRECTION
RCL6	36 06	F = 40,000 lbs
-	-45	
RCL7	36 07	
÷	-24	
STOE	35 15	
GTOe	22 16 15	
*LBL5	21 05	COMPUTE RANGE
RCLB	36 12	W/O TERMP CORRECTIO
RCL8	36 08	F = 50,000 lbs
-	-45	
RCL9	36 09	
÷	-24	
STOE	35 15	
*LBLe	21 16 15	
RCLB	36 12	COMPUTE TEMP
EEX	-23	CORRECTION
3	03	
÷	-24	
2	02	
CHS	-22	
×	-35	
1	01	
5	05	
+	-55	
RCLC	36 13	
-	-45	
STOD	35 14	
X≠0?	16-42	IF TEMP = STD
GTOd	22 16 14	DISPLAY RANGE
RCLE	36 15	
DSP0	-63 00	
RTN	24	
*LBLd	21 16 14	
.	-62	
0	00	TEMP ≠ STD
0	00	DISPLAY RANGE
2	02	
×	-35	
RCLE	36 15	
×	-35	
CHS	-22	
RCLE	36 15	
+	-55	
DSP0	-63 00	
R/S	51	END

E. Mathematical Analysis

A linear curve fit was performed using the HP-67/97 standard pack SD-03A program. Five "fits" were preformed. For a constant fuel weight, X represented the range and Y represented the altitude. Resulting outputs provided the following:

Fuel (pounds)	R^2	a	b
10,000	.998615613	-34,302.26904	122.9065370
20,000	.998739204	-35,846,62839	43.18615709
30,000	.999756772	-36,857.62093	27.00641670
40,000	.999823326	-38,068.60561	20.18918143
50,000	.999802631	-39,482.60271	16.50293201

For a constant fuel the range X could be obtained as follows:

$$X = \frac{Y - a}{b}$$

or

$$\text{Range} = \frac{\text{altitude - intercept (constant fuel)}}{\text{slope (constant fuel)}}$$

Temperature correction:

increase range 1% per 5°C above standard

decrease range 1% per 5°C below standard .

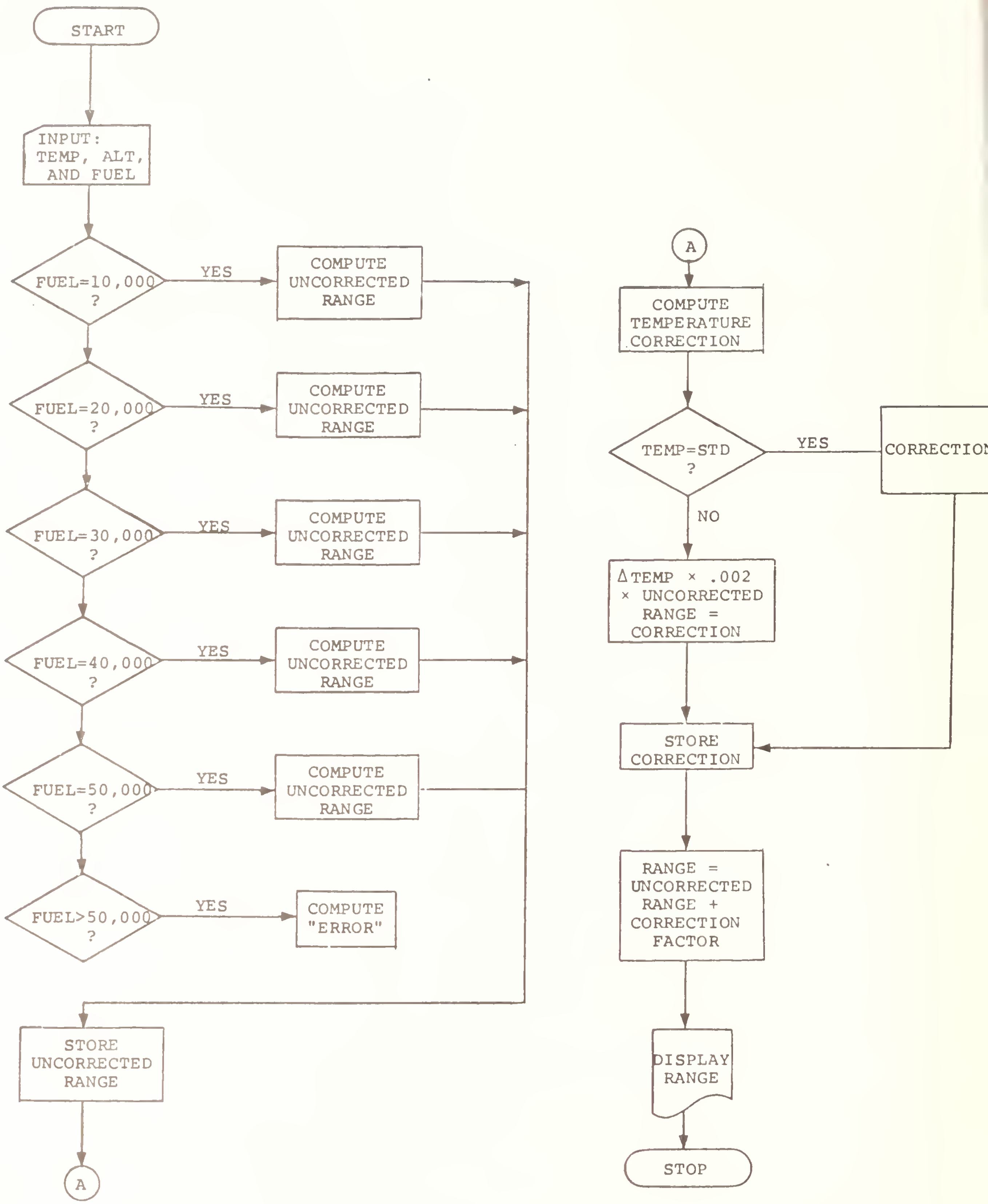
Based on a lapse rate of $-2°/1000ft$ the standard temperature for a specified altitude is obtained by:

$$-2 \times \frac{\text{altitude}}{1000} + 15 = \text{standard temperature (for altitude)}$$

Subtracting the input temperature from the computed standard temperature yielded the difference in °C from standard.

A correction factor could be obtained for each difference times .002/degree times uncorrected range. The range could then be computed by summing the correction factor and the uncorrected range value.

F. Program Flowchart



IV. MISSION PLANNING: P-3(B) AIRCRAFT CONFIGURATION "B"

LT M. D. Thomas.

A. Problem Statement

In order to carry out the various operational missions assigned the P3-B Aircraft, effective utilization of the platform is essential. All aspects of the mission must be carefully planned. Fuel planning directly influences endurance and the effectiveness of the mission. The NATOPS manual provides charts for this purpose. Two vital charts for planning are:

1. four engine maximum range operating table; used in proceeding to the operational area.
2. three engine loiter operating table; used while onstation for minimum fuel consumption.

The pilot or flight engineer enters with the aircraft's altitude and gross weight and finds the correct indicated airspeed (IAS) to fly.

This program is a user's program in that it translates these two charts onto an HP-67/97 magnetic card and allows calculation of IAS without the charts. Most missions are flown in configuration 'B' therefore the program presented here is for that case.

B. Operational Analysis

None.

C. Computational Algorithm

1. Enter altitude and gross weight in packed form: AAAAA.WWWW where AAAAA denotes the altitude in feet, and WWWW is the gross weight divided by 100,000. The leading zeroes, if any, in the value of WWW must be entered. For example, 18,000 feet and 76,500 pounds are entered as AAAAA = 18,000 and WWWW = 0765, that is 18,000.0765.

2. Compute the maximum range IAS or the three-engine loiter IAS.

D. HP-67/97 Calculator Program

1. User Instructions

Step	Instruction	Input	Key(s)	Output
1.	Enter program card			
2.	Enter altitude and gross weight. G.W. must be at least a three digit number. Compute four engine max range IAS.	ALT.GW	A	IAS (max range)
3a.	Enter altitude and gross weight. G.W. must be at least a three digit number. Compute three engine loiter range IAS	ALT.GW	B	IAS (Loiter)
b.	Optional: Compute the four engine max range IAS without re-entering the altitude and weight		R/S	IAS (max temp)

2. Sample Problems

	A Max Range IAS	B Loiter IAS
1. 130,000 lbs at 18,000 ft 18000.130	252	220
2. 86,000 lbs at 6,000 ft 6000.086*	241	175
3. 76,500 lbs at 10,000 ft 10000.0765	232	165
4. 50,000 lbs at 3,000 ft 3000.050	Error	Error
5. 130,000 lbs at 30,000 ft 30000.130	Error	Error

---

* Gross weight must be at least a three digit number (IE)

130,000	.130
76,000	.076
82,500	.0825

3. Program Storage Allocation and Listing

Registers

R0: altitude	S0:	RA:
R1: max range constant	S1:	⋮
R2: loiter airspeeds	S2:	
⋮	⋮	RE:
R9:	S9:	RI:

Initial Flag Status and Use:

0:	OFF, unused	2:	OFF, unused
1.	OFF, unused	3:	OFF, unused.

User Control Keys:

A:	Compute four engine IAS	a:	
B:	Compute three engine IAS	b:	
C:		c:	
D:		d:	
E:		e:	

001	*LBLA	21 11	UNPACK ALTITUDE AND GROSS WEIGHT
002	DSP0	-63 00	
003	ENT↑	-21	
004	INT	16 34	
005	STO0	35 00	
006	R↓	-31	
007	FRC	16 44	
008	EEX	-23	
009	3	03	
010	×	-35	
011	2	02	STORE MAX RANGE CONSTANT
012	.	-62	
013	7	07	
014	STO1	35 01	
015	X⇄Y	-41	
016	2	02	STORE LOITER CONSTANT
017	2	02	
018	0	00	
019	STO2	35 02	
020	R↓	-31	SET INITIAL GROSS WEIGHT BRACKET
021	1	01	
022	2	02	
023	7	07	
024	.	-62	
025	5	05	
026	GTO4	22 04	
027	*LBLB	21 12	COMPUTE LOITER IAS
028	GSBA	23 11	
029	RCL2	36 02	
030	RTN	24	CHECK ALTITUDE RESTRICTIONS
031	*LBL1	21 01	
032	GSB7	23 07	
033	R↓	-31	
034	1	01	
035	1	01	
036	7	07	
037	.	-62	

038	5	05	COMPUTE MAX RANGE IAS
039	X≤Y?	16-35	
040	GSB6	23 06	
041	RCL0	36 00	
042	RCL1	36 01	
043	EEX	-23	
044	5	05	
045	×	-35	
046	-	-45	
047	EEX	-23	
048	3	03	
049	÷	-24	
050	CHS	-22	
051	RTN	24	
052	*LBL4	21 04	CONDITIONAL TEST OF GROSS WEIGHT CATEGORY INCREMENTS CONSTANTS
053	X≤Y?	16-35	
054	GTO1	22 01	
055	.	-62	
056	0	00	
057	3	03	
058	ST-1	35-45 01	
059	R↓	-31	
060	5	05	
061	ST-2	35-45 02	
062	-	-45	
063	GTO5	22 05	
064	RTN	24	
065	*LBL5	21 05	CONDITIONAL TEST OF GROSS WEIGHT CATEGORY INCREMENTS CONSTANTS
066	X≤Y?	16-35	
067	GTO1	22 01	
068	.	-62	
069	0	00	
070	2	02	
071	ST-1	35-45 01	
072	R↓	-31	
073	5	05	
074	ST-2	35-45 02	

075	-	-45	
076	GTO4	22 04	
077	RTN	24	
078	*LBL6	21 06	CHECKS ALTITUDE IF ILLEGAL, ERROR DISPLAY
079	RCL0	36 00	
080	2	02	
081	8	08	
082	EEX	-23	
083	3	03	
084	X>Y?	16-34	
085	RTN	24	
086	X⇄Y	-41	
087	PRTX	-14	
088	GTOC	22 13	
089	*LBL7	21 07	CHECKS GROSS WEIGHT CATEGORY IF ILLEGAL, ERROR DISPLAY
090	R↓	-31	
091	7	07	
092	2	02	
093	.	-62	
094	5	05	
095	X≤Y?	16-35	
096	RTN	24	
097	X⇄Y	-41	
098	PRTX	-14	
099	GTOC	22 13	
100	RTN	24	
101	R/S	51	

E. Computational Analysis

Using the HP-67 standard curve fitting program, a good linear fit was obtained on the four engine maximum range data. There is a linear relationship between altitude and indicated airspeed for each gross weight category. The coefficient of determination was equal to 1.00 in all cases, indicating a good fit. The following equations were used; loiter airspeeds are constant for each category.

G.W. (1000 lbs)	Max range IAS	Loiter IAS
132.5-127.5	$y = 270 - x/1000$	220
127.5-122.5	$y = 267 - x/1000$	215
122.5-117.5	$y = 265 - x/1000$	210
117.5-112.5	$y = 262 - x/1000$	205
112.5-107.5	$y = 260 - x/1000$	200
107.5-102.5	$y = 257 - x/1000$	195
102.5- 97.5	$y = 255 - x/1000$	190
97.5- 92.5	$y = 252 - x/1000$	185
92.5- 87.5	$y = 250 - x/1000$	180
87.5- 82.5	$y = 247 - x/1000$	175
82.5- 77.5	$y = 245 - x/1000$	170
77.5- 72.5	$y - 242 - x/1000$	165

x = altitude in feet and y = maximum range IAS.

V. USER-CONTROLLED SIMULATION OF APPROACH AND LANDING FOR THE P-3 AIRCRAFT by LT J. Aiken

A. Problem Statement

This program simulates an aircraft approach and landing. Specifically, it is a time-step simulation of the final five miles of a precision approach for a Lockheed P-3 ORION aircraft. The simulation is user-controlled which allows the user to act as pilot and make the decisions which control the movement of the airplane during its final approach phase. The purpose of the program is to simulate accurately the flight of the aircraft and display to the operator his rate of movement and position resulting from his manipulation of the controls.

B. Operation Analysis

Relevant information on the airfield is as follows:

Runway	8000 ft (length
	200 ft (width)
	180 degrees magnetic heading
	SEA LEVEL elevation
Approach	TOUCHDOWN POINT 1000ft beyond approach threshold
	GLIDE SLOPE 2.83 degrees
	2 min 18 sec time required at 135 kts ground speed
	FINAL APPROACH FIX: 5 miles, 1500ft (starting point)

The aircraft weighs 90,000 lbs with approach speeds 135/121 kts (approach flaps/land flaps). Note however that no provision is made for changing the gear/flap configuration so it is essentially an "approach-flap" landing. The simulation starts with the aircraft in motion: 1500 feet MSL, 135 kts IAS, 650 ft/min descent rate, landing gear down and approach flaps. The simulation allows the user to select horsepower settings, nose attitude, heading, wind direction, wind velocity, and time interval. At the end of a time interval the simulation is halted and the critical flight information is displayed to the operator, allowing him to alter controlling parameters and continue the flight. The simulation continues in this manner until the aircraft lands. Vital landing parameters are displayed and the simulation is complete. The simulation realistically responds to control changes provided the aircraft is flown in a somewhat reasonable fashion. Extreme deviations and maneuvers other than those required during an approach are not designed into the program.

C. Computational Algorithm

1. Initialize the aircraft at the starting point.
2. Input time step, wind direction and wind velocity.
3. Input horsepower, nose attitude, heading, and number of time steps desired.
4. Compute course deviation.
5. Compute horizontal acceleration.
6. Compute vertical acceleration.
7. Compute final velocity and average vertical velocity.
8. Compute altitude.
9. Compute final and average horizontal velocity.
10. Compute distance remaining based on ground speed.
11. Compute glide slope height and deviation from glide slope.
12. Check altitude less than 0.
13. DSZ (number of time steps is the counter) GTO 4 above.
14. Display approach parameters after completing desired time steps.
15. Display landing parameters upon landing.
16. Clear primary and secondary registers, GTO 1 for new problem.

D. HP-67 Calculator Program

1. User Instructions

Step	Instruction	Input	Key(s)	Output
1.	Enter program card			
2.	Clear primary and secondary registers			
3.	Initialize		f e	30384
4.	Enter time step	seconds	STO C	
5.	Enter wind direction	degrees	STO D	
6.	Enter wind velocity	knots	STO E	
7.	Enter horsepower	HP	A	HP
8.	Enter nose attitude	± degrees	B	Nose attitude
9.	Enter heading	degress	C	Heading
10.	Enter number of time steps	integer	D	flashing
	Output			
	Altitude. Airspeed (Packed)			Alt. airspeed
	Descent rate (ft/min)		R/S	Descent rate
	Above (+) or below (-) glide slope		R/S	Feet hi/lo
	Distance to go Airborne (miles) Landed (feet)		R/S	± Distance to go
	Right (+) or left (-) of of course		R/S	ft right/left
11.	If altitude GT zero go to Step 4; make new entries only if change desired. Step 10 must be re-entered.			

2. Sample Problem

Input							Output					
Time Step	Wind Direction	Wind Velocity	Horsepower	Nose Attitude	Heading	Steps	Altitude	Airspeed	Descent Rate	Above/Below Glide Slope	Distance To Go	Right/Left of Course
20	210	20	800	1.5	185	1	1295	133	-570	-1	4.4	63
					183	1	1108	132	-552	2.4	3.7	-34
			790		184	1	922	132	-560	6.9	3.1	-51
			775		185	1	732	132	-580	6.3	2.4	11
					184	1	538	132	-584	1.6	1.8	- 6
10			790		184	1	441	132	-575	0	1.5	-14
					184.8	1	346	132	-574	-1	1.2	9
			795	2	184.3	1	252	131	-555	-1	.85	13
5			700	3.5	184	1	205	129	-559	-1	.7	8
				4		1	159	127	-552	-2	.5	4
					184.1	1	113	127	-552	-2	.4	2
				4.3	184	1	67	127	-547	-3	.24	- 2
				4.8		1	22	126	-538	-3	.08	- 7
2				6		1	4	126	-530	-3	.02	- 8
				8	182.5	1	0	125	-516	-2	40	-22

Sample Problem Keystroke Sequence

```
            GSBe
30384.00     ***
   20.00    STOC
  210.00    STOD
   20.00    STOE
  800.00    GSBA
    1.50    GSBE
  185.00    GSBC
    1.00    GSBD
1255.133     ***
             R/S
 -570.00     ***
             R/S
   -1.62     ***
             R/S
    4.35     ***
             R/S
   62.92     ***

  183.00    GSBC
    1.00    GSBD
1168.132     ***
             R/S
 -552.06     ***
             R/S
    2.42     ***
             R/S
    3.71     ***
             R/S
  -34.34     ***
  790.00    GSBA
  164.00    GSBC
    1.00    GSBD
 922.132     ***
             R/S
 -560.40     ***
             R/S
    6.69     ***
             R/S
    3.63     ***
             R/S
  -51.46     ***
  775.00    GSBA
  185.00    GSBC
    1.00    GSBD
 732.132     ***
             R/S
```

```
 -580.06     ***
             R/S
    6.33     ***
             R/S
    2.44     ***
             R/S
   11.46     ***
  184.00    GSBC
    1.00    GSBD
 536.132     ***
             R/S
 -584.02     ***
             R/S
    1.61     ***
             R/S
    1.60     ***
             R/S
   -5.66     ***
   10.00    STOD
  790.00    GSBA
  184.00    GSBC
    1.00    GSBD
 441.132     ***
             R/S
 -575.41     ***
             R/S
   -0.35     ***
             R/S
    1.49     ***
             R/S
  -14.22     ***
  184.00    GSBC
    1.00    GSBD
 346.132     ***
             R/S
 -573.69     ***
             R/S
   -1.40     ***
             R/S
    1.17     ***
             R/S
    9.24     ***
  795.00    GSBA
    2.00    GSBE
  184.30    GSBC
    1.00    GSBD
 252.131     ***
```

Sample Problem Keystroke Sequence (cont.)

```
             R/S
  -555.34    ***
             R/S
    -1.25    ***
             R/S
     0.65    ***
             R/S
    12.65    ***
     5.00 STOC
   700.00 GSBa
     3.50 GSBE
   184.00 GSBC
     1.00 GSBC
  205.129    ***
             R/S
  -559.51    ***
             R/S
    -1.27    ***
             R/S
     0.69    ***
             R/S
     8.41    ***
     4.00 GSBE
     1.00 GSBC
  159.127    ***
             R/S
  -552.84    ***
             R/S
    -1.85    ***
             R/S
     0.54    ***
             R/S
     4.13    ***
   184.10 GSBC
     1.00 GSBC
  113.127    ***
             R/S
  -551.51    ***
             R/S
    -2.35    ***
             R/S
     0.35    ***
             R/S
     1.85    ***
     4.30 GSBE
   184.00 GSBC
```

```
     1.00 GSBC
   67.127    ***
             R/S
  -546.74    ***
             R/S
    -2.72    ***
             R/S
     0.24    ***
             R/S
    -2.43    ***
     4.00 GSBE
     1.00 GSBC
   22.126    ***
             R/S
  -538.29    ***
             R/S
    -2.76    ***
             R/S
     0.08    ***
             R/S
    -6.71    ***
     2.00 STOC
     6.00 GSBE
     1.00 GSBC
    4.126    ***
             R/S
  -530.41    ***
             R/S
    -2.61    ***
             R/S
     0.02    ***
             R/S
    -8.42    ***
     8.00 GSBE
   182.50 GSBC
     1.00 GSBC
    0.125    ***
             R/S
  -515.64    ***
             R/S
    -2.24    ***
             R/S
    40.50    ***
             R/S
   -22.16    ***
```

3. Program Storage Allocations and Program Listing

Registers:

R0: Right/left of course
S1: Hortizonal force

R1: Horsepower
S2: Vertical force

R2: Nose attitude

R3: Horizontal velocity

R4: Vertical velocity

R5: Altitude

R6: Horizontal acceleration

R7: Vertical acceleration

R8: Distance remaining

R9: Glide Slope Altitude

RA: DELH

RB: Heading

RC: Time step

RD: Wind direction

RE: Wind velocity

RI: Number of time steps

Initial Flag Status:

0: OFF, Unused

1: OFF, Unused

2: OFF, Set ON upon landing

3: OFF, Unused

User Control Keys:

A: Horsepower

B: Nose attitude

C: Heading

D: Time steps, start computation

E:

a:

b:

c:

d:

e: Initialize

Step	Key	Code	Comment
001	*LBLA	21 11	
002	STO1	35 01	INPUT
003	R/S	51	
004	*LBLB	21 12	HORSEPOWER
005	STO2	35 02	NOSE ATTITUDE
006	R/S	51	
007	*LBLC	21 13	HEADING
008	STOB	35 12	
009	R/S	51	
010	*LBLe	21 16 15	
011	2	02	
012	2	02	
013	8	08	
014	STO3	35 03	
015	1	01	
016	1	01	
017	CHS	-22	INITIALIZE
018	STO4	35 04	
019	1	01	SPEEDS
020	5	05	
021	0	00	ALTITUDE
022	0	00	
023	STO5	35 05	DISTANCE
024	3	03	
025	6	06	
026	3	03	
027	6	06	
028	4	04	
029	STO8	35 08	
030	R/S	51	
031	*LBLD	21 14	
032	STOI	35 46	
033	*LBL8	21 08	
034	RCLB	36 12	
035	1	01	
036	8	08	
037	0	00	
038	-	-45	
039	SIN	41	
040	2	02	
041	3	03	COMPUTE
042	0	00	
043	x	-35	DRIFT RATES
044	1	01	
045	8	08	AND
046	0	00	
047	RCLD	36 14	COURSE
048	-	-45	
049	SIN	41	DEVIATION
050	RCLE	36 15	
051	1	01	
052	.	-62	
053	6	06	
054	9	09	
055	x	-35	
056	x	-35	
057	+	-55	
058	RCLC	36 13	
059	x	-35	
060	ST+0	35-55 00	

Step	Key	Code	Comment
061	GSB1	23 01	COMPUTE HORIZONTAL ACCEL
062	STO6	35 06	
063	GSB2	23 02	COMPUTE VERTICAL ACCEL
064	STO7	35 07	
065	RCL4	36 04	COMPUTE
066	RCL7	36 07	
067	RCLC	36 13	VERTICAL
068	x	-35	
069	ST+4	35-55 04	VELOCITY
070	X⇄Y	-41	
071	RCL4	36 04	
072	+	-55	
073	2	02	
074	÷	-24	COMPUTE
075	RCLC	36 13	
076	x	-35	ALTITUDE
077	ST+5	35-55 05	
078	RCL3	36 03	COMPUTE
079	RCL6	36 06	HORIZONTAL
080	RCLC	36 13	
081	x	-35	VELOCITY
082	ST+3	35-55 03	
083	X⇄Y	-41	
084	RCL3	36 03	
085	+	-55	
086	2	02	
087	÷	-24	COMPUTE
088	RCLD	36 14	
089	1	01	GROUND
090	8	08	
091	0	00	SPEED
092	-	-45	
093	ABS	16 31	AND
094	COS	42	
095	RCLE	36 15	DISTANCE
096	1	01	
097	.	-62	TO GO
098	6	06	
099	9	09	
100	x	-35	
101	x	-35	
102	-	-45	
103	RCLC	36 13	
104	x	-35	
105	CHS	-22	
106	RCL8	36 08	COMPUTE
107	+	-55	GLIDE-SLOPE
108	STO8	35 08	HEIGHT
109	.	-62	AND
110	0	00	DEVIATION
111	4	04	FROM
112	9	09	GLIDE-SLOPE
113	x	-35	
114	STO9	35 09	
115	CHS	-22	
116	RCL5	36 05	
117	+	-55	
118	STOA	35 11	
119	RCL5	36 05	
120	X<0?	16-45	CHECK ALT < 0
121	GTO9	22 09	
122	DSZI	16 25 46	LOOP CONTROL
123	GTO8	22 08	

124	*LBL7	21 07	ALTITUDE DECIMAL AIRSPEED (PACKED)
125	RCL5	36 05	
126	INT	16 34	
127	RCL3	36 03	
128	1	01	
129	6	06	
130	8	08	
131	8	08	
132	÷	-24	
133	+	-55	
134	DSP3	-63 03	
135	R/S	51	
136	DSP2	-63 02	RATE OF DESCENT
137	RCL4	36 04	
138	6	06	
139	0	00	
140	×	-35	
141	R/S	51	
142	RCLA	36 11	HIGH/LOW
143	R/S	51	
144	RCL8	36 08	DISTANCE TO GO (FEET IF LANDED)
145	F2?	16 23 02	
146	GTO5	22 05	
147	6	06	
148	0	00	
149	7	07	
150	7	07	
151	÷	-24	
152	*LBL5	21 05	
153	R/S	51	
154	RCL0	36 00	LEFT/RIGHT
155	R/S	51	
156	*LBL9	21 09	LANDED... ADJUST DISTANCE TO ZERO ALT.
157	RCL8	36 08	
158	RCL5	36 05	
159	CHS	-22	
160	2	02	
161	0	00	
162	×	-35	
163	+	-55	
164	STO8	35 08	
165	0	00	SET ALT = 0
166	STO5	35 05	
167	SF2	16 21 02	SET FLAG 2
168	GTO7	22 07	

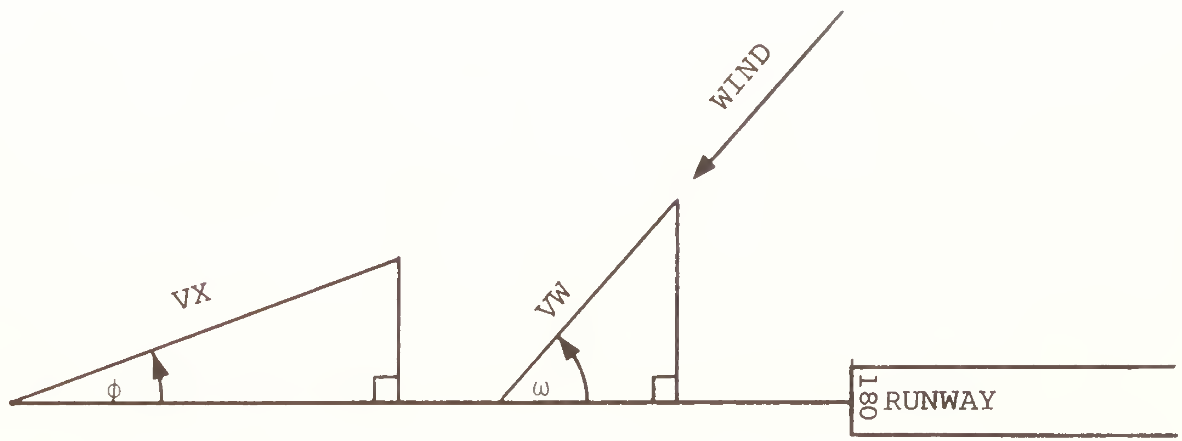
169	*LBL1	21 01	COMPUTE HORIZONTAL FORCE
170	RCL2	36 02	
171	.	-62	
172	4	04	
173	CHS	-22	
174	×	-35	
175	EEX	-23	
176	3	03	
177	CHS	-22	
178	RCL1	36 01	
179	×	-35	
180	+	-55	
181	.	-62	
182	4	04	
183	-	-45	
184	P⇄S	16-51	COMPUTE HORIZONTAL ACCELERATION
185	RCL1	36 01	
186	X⇄Y	-41	
187	STO1	35 01	
188	X⇄Y	-41	
189	-	-45	
190	P⇄S	16-51	
191	RCL6	36 06	
192	5	05	
193	÷	-24	
194	+	-55	
195	RTN	24	
196	*LBL2	21 02	COMPUTE VERTICAL FORCE
197	.	-62	
198	0	00	
199	0	00	
200	1	01	
201	RCL1	36 01	
202	×	-35	
203	.	-62	
204	8	08	
205	-	-45	
206	RCL2	36 02	
207	.	-62	
208	0	00	
209	5	05	
210	×	-35	
211	+	-55	
212	P⇄S	16-51	COMPUTE VERTICAL ACCELERATION
213	RCL2	36 02	
214	X⇄Y	-41	
215	STO2	35 02	
216	X⇄Y	-41	
217	-	-45	
218	P⇄S	16-51	
219	RCL7	36 07	
220	5	05	
221	÷	-24	
222	+	-55	
223	RTN	24	
224	R/S	51	

E. Mathematical Analysis

The variable names used are:

VX	Horizontal Velocity
VY	Vertical Velocity
VW	Wind Velocity
DRA	Drift Rate Due to A/C HDG
DRW	Drift Rate Due to Wind
TDR	Total Drift Rate
GS	Ground Speed
AX	Horizontal Velocity
AY	Vertical Velocity
D	Distance to go
H	Glide Slope Altitude
DELH	Deviation from Glide Slope
ALT	Altitude
t	Time Step

COURSE DEVIATION COMPUTATION



ϕ = HDG - 180

ω = 180 - Wind Dir

ϕ is negative (on this diagram)

DRA = VX sin ϕ

DRW = VW sin ω

TRD = DRA + DRW

TOTAL DRIFT = TDR(t)

DISTANCE, GROUNDSPEED, GLIDE SLOPE

$$\text{Avg HOR Velocity} = \frac{1}{2}(VX_0 + VX_1) \quad \text{(for one time step)}$$

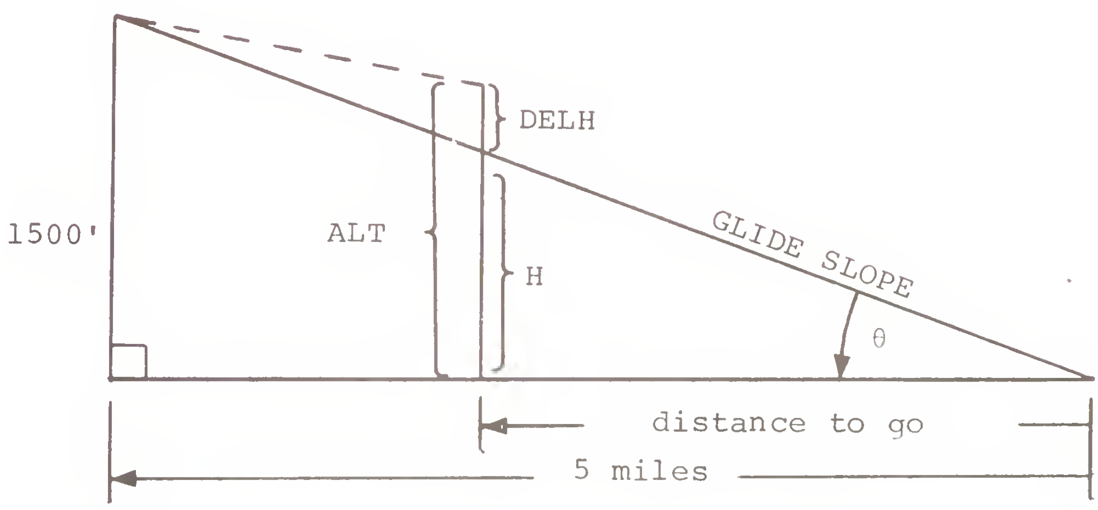
$$\text{Distance travelled} = GS(t)$$

$$GS = VX - VW \cos|\text{wind dir} - 180|$$

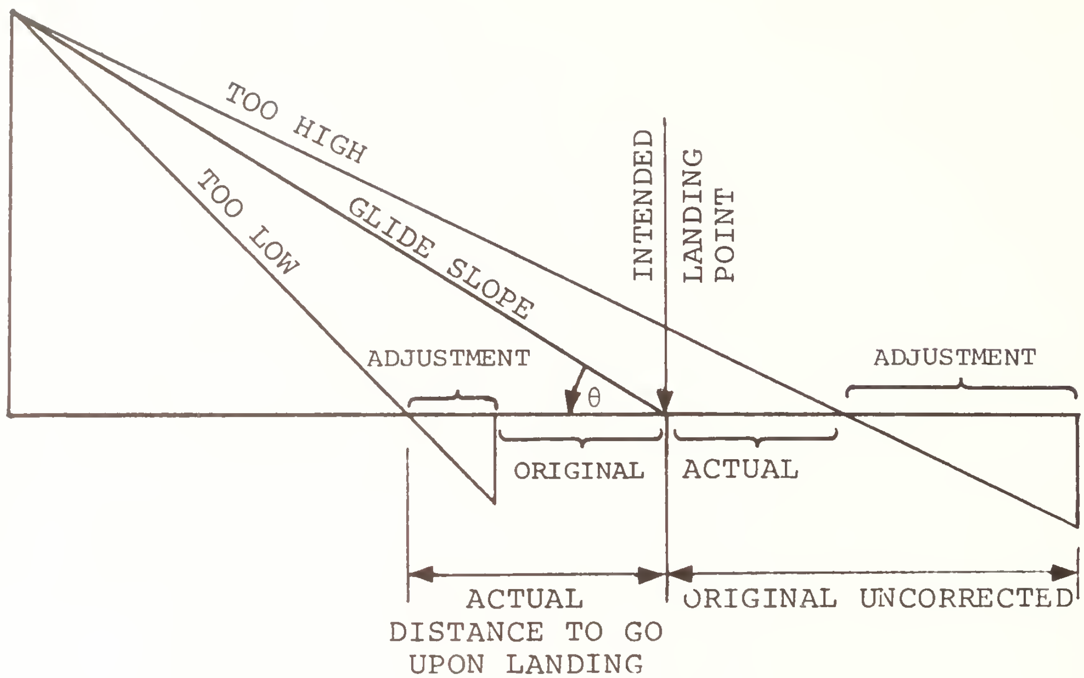
$$D = \text{prior remaining} - \text{distance travelled}$$

$$H = D \tan\theta$$

$$DELH = ALT - H$$



LANDING DISTANCE ADJUSTMENT



Two situations are illustrated above.

1. Aircraft above glide slope (lands long).

2. Aircraft below glide slope (lands short).

An adjustment is required because the aircraft is allowed to descend until the end of the time step. On the landing time step the aircraft will have descended below zero altitude. The adjustment in either case is: adjustment = $H/(\tan\theta)$.

FORCES, ACCELERATIONS, AND VELOCITIES

$FX = -\ \theta + .001HP - .4$

$FY = .001HP - .8 + .05\theta$

$AX' = (FX^1 - FX^0) + .2AX^0$

$AY' = (FY^1 - FY^0) + .2AY^0$

$VX_1 = VX_0 + AX(t)$

$VY_1 = VY_0 + AX(t)$

FX = horizontal force

FY = vertical force

θ = nose attitude

Superscripts denote different time steps

Subscripts denote start and end of a time step.

Although the author is not an aeronautical engineer, it was felt that his understanding of the basic laws of physics complemented by considerable pilot experience would serve sufficiently to accomplish the goals of this project. The formulae for force and acceleration were arrived at after testing several trial formulae on experienced P-3 pilots. The unanimous opinion was that the current program enables the simulation to closely model the actual flight characteristics of the aircraft.

VI. FLIGHT CREW MANAGEMENT USING THE HP-97 by LT Kenneth W. Peters

A. Problem Statement

A flight crew's most recent landing day and time is known. Using requirements for crew rest and postflight and preflight duration, compute when the flight crew will be available for take-off again. For planning and scheduling purposes, list crews in order of availability. For required onstation times compute takeoff, onstation, offstation and landing times for a given number of flights. Determine if flight crews will be available to meet this schedule.

B. Operational Analysis

When planning an operation requiring scheduling of several flight crews, crew availability must be considered. Accurate and easily understood crew records are necessary to meet both operational and safety requirements.

C. Computational Algorithm

1. Flight crew availability
    a. Enter required postflight to preflight crew rest time.
    b. Enter crew number and their most recent landing day and time.
    c. Compute the crew's earliest possible takeoff day and time using one hour for postflight and three hours for preflight.

2. List crews in order of availability

   a. Enter number of flight crews = N.

   b. Compare crew N availability with crew (N-1) availabili Store crew number and availability of crew which can take-off soonest. Compare this crew with crew (N-x), (x = 1,2,3,4,...,(N-1)), and store the crew which is available the soonest.

   c. When most available crew has been determined, add 10,000 days to its takeoff availability date and increment counter.

   d. Repeat Steps b and c until all crews have been listed, i.e. counter = N.

   e. Restore crew availability data by subtracting 10,000 days from each crew's availability date.

3. Operational schedule.

   a. Enter required number of flights from a particular base and the gap (±) desired for onstation coverage.

   b. Compute takeoff day and time, onstation day and time, offstation day and time, and landing day and time.

   c. Check number of flights versus counter. Repeat b as required.

   d. Compare print-out of required takeoff times with crew availability listing (see Part 2 above).

D. HP-67/97 Calculator Program

1. User Instructions

Step	Instruction	Input	Key(s)	Output
1.	Enter program card			
2.	Compute flight crew availability			
a.	Enter crew rest time	HH.MM	fA	
b.	Enter crew rest data:			
	Crew #	$1 \leq \# \leq 19$	↑	
	Postflight time	HH.MM	↑	
	Preflight time	HH.MM	↑	
	Landing Julian day and time	DAY.HHMM	A	Crew # Availability DAY.HHM
	NOTE: If change in year will occur, Julian date can be entered as: YRDAY.HHMM			
3.	List crews in order of availability. Enter # of crews = N	$1 \leq N \leq 19$	B	Crew # Availability DAY.HHMM (repeated N times)
4.	Compute operational flight schedule			
a.	Enter # of flights and gap in onstation coverage:			
	# of flights = N	$1 \leq N \leq 25$	↑	
	gap in coverage	HH.MM	fC	
b.	Compute schedule			
	onstation day and time	DAY.HHMM	↑	
	one-way transit time	HH.MM	↑	
	mission time	HH.MM	C	Flight # Takeoff DAY.HHMM Onsta DAY.HHMM Offsta DAY.HHMM Land DAY.HHMM
5.	(Optional) Store crew availability data on magnetic card: Set W/PRGM-RUN switch to RUN		f W/DATA	
	To reload data: Set W/PRGM-RUN switch to RUN and load data card obtained above			

2. Sample Problem

a. Input the following data:

Crew rest = 15 hours

Crew #	Postflight	Preflight	Landing
1	1	3	2.2330
2	1	3	1.1200
3	1	3	2.1800
4	1	3	1.1532

b. Compute crew availability.

Answer:

Crew #	Availability
1	3.1830
2	2.0700
3	3.1300
4	2.1032

c. Input the following crew status:

Crew #	(in) Register #	Availability
1	1	3.1830
2	2	2.0700
3	3	3.1300
4	4	2.1032
5	5	2.0958
6	6	2.2020
7	7	3.1930
8	8	5.0730
9	9	4.0930
10	10	6.0430
11	11	4.0345
12	12	2.0730
13	13	8.0130
14	14	5.0315
15	15	3.0545
16	16	3.1300
17	17	9.0955
18	18	2.1030

d. Output listing of crews in order of availability.

Answer: 2, 12, 5, 18, 4, 6, 15, 3, 16, 1, 7, 11, 9, 14, 8, 10, 13, 17.

e. Develop an operational flight schedule consisting of six (6) flights with a zero (0) gap in onstation coverage. The first onstation time is 2.1830, it takes one (1) hour and fifty (50) minutes for a one-way transit, and a flight from this base has nine (9) hours of total mission time.

Answer:

Flight #	Takeoff	Onsta	Offsta	Land
1	2.1640	2.1830	2.2350	3.0140
2	2.2200	2.2350	3.0510	3.0700
3	3.0320	3.0510	3.1030	3.1220
4	3.0840	3.1030	3.1550	3.1740
5	3.1400	3.1550	3.2110	3.2300
6	3.1920	3.2110	4.0230	4.0420

f. Develop an operational flight schedule consisting of two (2) flights with a two (2) hour gap in onstation coverage. The first onstation time is 2.1830, with the same transit and mission time as in (e).

Answer:

Flight #	Takeoff	Onsta	Offsta	Land
1	2.1640	2.1830	2.2350	3.0140
2	3.0000	3.0150	3.0710	3.0900

g. Develop an operational flight schedule with the same parameters as in (f), except with a negative two (-2) hour gap in onstation coverage.

Answer:

Flight #	Takeoff	Onsta	Offsta	Land
1	2.1640	2.1830	2.2350	3.0140
2	2.2100	2.2250	3.0410	3.0600

h. With one flight from Base A with the first onstation time as above and one flight from Base B to the same operational area, compute the flight schedule. One-way transit time from Base B is two (2) hours, but mission time is now ten (10) hours. Zero (0) gap in coverage is desired.

Answer:

Flight #	Takeoff	Onsta	Offsta	Land
1	2.1640	2.1830	2.2350	3.0140
2	2.2120	2.2350	3.0450	3.0720

i. With four (4) flight crews with crew availability as follows:

Crew #	Availability
1	3.1830
2	2.0700
3	3.1300
4	3.1700

determine if they are available to meet an operational flight schedule with the following conditions:

4 flights

2 hour gap in onstation coverage

3.0100 is the first onstation time

1 hour and 30 minute one-way transit

10 hour mission time

Answer: No, a crew is not available to meet the second scheduled takeoff time.

Examples a, b

```
15.0000 GSBe
 1.0000 ENT↑
 1.0000 ENT↑
 3.0000 ENT↑
 3.2230 GSBa
     1.   ***
 3.1630   ***

 2.0600 ENT↑
 1.0000 ENT↑
 3.0000 ENT↑
 1.1200 GSBa
     2.   ***
 2.0700   ***
```

```
3.0000 ENT↑
1.0000 ENT↑
3.0000 ENT↑
2.1800 GSBa
    3.   ***
3.1300   ***

4.0000 ENT↑
1.0000 ENT↑
3.0000 ENT↑
1.1532 GSBa
    4.   ***
2.1032   ***
```

Examples c, d

16.0000 GSBE 2. *** 2.0700 ***	15. *** 3.0545 ***	9. *** 4.0930 ***
12. *** 2.0730 ***	3. *** 3.1300 ***	14. *** 5.0315 ***
5. *** 2.0958 ***	16. *** 3.1300 ***	8. *** 5.0730 ***
18. *** 2.1030 ***	1. *** 3.1630 ***	10. *** 6.0430 ***
4. *** 2.1032 ***	7. *** 3.1930 ***	13. *** 8.0130 ***
6. *** 2.2020 ***	11. *** 4.0345 ***	17. *** 9.0955 ***

Example e

```
     6.0000 ENT↑
     0.0000 GSB C

     2.1830 ENT↑
     1.5000 ENT↑
     9.0000 GSB C
 1.00000000   ***
     2.1640   ***
     2.1830   ***
     2.2350   ***
     3.0140   ***

 2.00000000   ***
     2.2200   ***
     2.2350   ***
     3.0510   ***
     3.0700   ***

 3.00000000   ***
     3.0320   ***
     3.0510   ***
     3.1030   ***
     3.1220   ***

 4.00000000   ***
     3.0840   ***
     3.1030   ***
     3.1550   ***
     3.1740   ***

 5.00000000   ***
     3.1400   ***
     3.1550   ***
     3.2110   ***
     3.2300   ***

 6.00000000   ***
     3.1920   ***
     3.2110   ***
     4.0230   ***
     4.0420   ***
```

Example f

```
     2.0000 ENT↑
     2.0000 GSB C

     2.1930 ENT↑
     1.5000 ENT↑
     9.0000 GSB C
 1.00000000   ***
     2.1640   ***
     2.1830   ***
     2.2350   ***
     3.0140   ***

 2.00000000   ***
     3.0000   ***
     3.0150   ***
     3.0710   ***
     3.0900   ***
```

Example g

```
     2.0000 ENT↑
    -1.0000 GSB C

     2.1830 ENT↑
     1.5000 ENT↑
     9.0000 GSB C
 1.00000000   ***
     2.1640   ***
     2.1830   ***
     2.2350   ***
     3.0140   ***

 2.00000000   ***
     2.2100   ***
     2.2250   ***
     3.0410   ***
     3.0600   ***
```

Example h

```
    1.0000 ENT↑
    0.0000 GSBc

    3.1830 ENT↑
    1.5000 ENT↑
    9.0000 GSBC
1.00000000  ***
    2.1640  ***
    2.1630  ***
    2.2350  ***
    3.0140  ***

    1.0000 ENT↑
    0.0000 GSBc

    2.2350 ENT↑
    2.3000 ENT↑
   10.0000 GSBC
1.00000000  ***
    2.2120  ***
    2.2350  ***
    3.0450  ***
    3.0720  ***
```

Example i

```
 4.0000 GSB6
     2.  ***
 3.0700  ***



     3.  ***
 3.1300  ***



     4.  ***
 3.1700  ***



     1.  ***
 3.1830  ***
```

```
    4.0000 ENT↑
    2.0000 GSBc
    3.0100 ENT↑
    1.3000 ENT↑
   10.0000 GSBC
1.00000000  ***
    2.2330  ***
    3.0100  ***
    3.0800  ***
    3.0930  ***

2.00000000  ***
    3.0530  ***
    3.1000  ***
    3.1700  ***
    3.1830  ***

3.00000000  ***
    3.1730  ***
    3.1900  ***
    4.0200  ***
    4.0330  ***

4.00000000  ***
    4.0230  ***
    4.0400  ***
    4.1100  ***
    4.1230  ***
```

3. Program Storage Allocation and Listing

Registers:

R0:	Crew rest	S0:	Crew 10 availability
R1:	Crew 1 availability	S1:	Crew 11 availability
R2:	Crew 2 availability	S2:	Crew 12 availability
R3:	Crew 3 availability	S3:	Crew 13 availability
R4:	Crew 4 availability	S4:	Crew 14 availability
R5:	Crew 5 availability	S5:	Crew 15 availability
R6:	Crew 6 availability	S6:	Crew 16 availability
R7:	Crew 7 availability	S7:	Crew 17 availability
R8:	Crew 8 availability	S8:	Crew 18 availability
R9:	Crew 9 availability	S9:	Crew 19 availability

RA: Landing data; # of crews

RB: First onsta time; crew #; takeoff time

RC: Mission time; flight counter

RD: Gap between onstation periods; ith availability

RE: Flight counter; crew counter; one-way transit

Initial Flag Status and Use

0:	Unused	2:	OFF, day correction
1:	Unused	d:	OFF, hour correction

User Controlled Keys

A: Crew # ↑, postflight ↑, preflight ↑, land ⇒ compute availability

B: crew # ↑; compute listing

C: onsta ↑, one-way transit ↑, mission time ⇒ flight schedule

D: unused

E: unused

a:	crew rest	d:	unused
b:	unused	e:	unused
c:	flight ↑, gap		

001	*LBLc	21 16 13	Input and Store # of flights and desired gap
002	STOD	35 14	
003	R↓	-31	
004	STOA	35 11	
005	RTN	24	
006	*LBLC	21 13	
007	CF2	16 22 02	Compute flight schedule
008	CF3	16 22 03	Input and store onstation day and time
009	STOC	35 13	Input and store one-way transit time
010	R↓	-31	Input and store total mission time
011	STOE	35 15	
012	R↓	-31	
013	STOB	35 12	
014	GSB0	23 00	
015	RCLE	36 15	Unpack ddd.hhmm to hh.mm
016	CHS	-22	
017	HMS+	16-55	Compute takeoff time
018	X<0?	16-45	Check for time greater than 24 hours
019	GSBe	23 16 15	
020	EEX	-23	
021	2	02	
022	÷	-24	
023	RCLB	36 12	
024	INT	16 34	
025	F3?	16 23 03	
026	GSBd	23 16 14	
027	+	-55	Takeoff ddd.hhmm
028	STOB	35 12	
029	1	01	
030	*LBL9	21 09	
031	STOI	35 46	Start loop
032	DSP0	-63 00	
033	PRTX	-14	Print flight #
034	RCLB	36 12	
035	F2?	16 23 02	
036	GSB2	23 02	
037	DSP4	-63 04	
038	PRTX	-14	Print takeoff ddd.hhmm
039	STOB	35 12	
040	STO0	35 00	
041	GSB0	23 00	
042	RCLE	36 15	
043	HMS+	16-55	Compute and print onstation ddd.hhmm
044	GSBD	23 14	
045	GSBE	23 15	
046	GSB0	23 00	
047	RCLE	36 15	
048	RCLE	36 15	
049	HMS+	16-55	
050	CHS	-22	
051	RCLC	36 13	
052	HMS+	16-55	
053	HMS+	16-55	Compute and print offstation ddd.hhmm
054	GSBD	23 14	
055	GSBE	23 15	
056	GSB0	23 00	
057	RCLE	36 15	
058	HMS+	16-55	Compute and print landing ddd.hhmm
059	GSBD	23 14	
060	GSBE	23 15	
061	SPC	16-11	

062	RCLB	36 12	
063	GSB0	23 00	
064	RCLC	36 13	
065	HMS+	16-55	
066	RCLE	36 15	
067	RCLE	36 15	
068	HMS+	16-55	
069	CHS	-22	
070	HMS+	16-55	Compute next takeoff ddd.hhmm
071	RCLD	36 14	
072	HMS+	16-55	
073	GSBD	23 14	
074	RCLB	36 12	
075	INT	16 34	
076	+	-55	
077	STOB	35 12	
078	RCLA	36 11	Increase counter
079	1	01	
080	+	-55	
081	1	01	
082	RCLI	36 46	
083	+	-55	
084	X=Y?	16-33	Check for exit from loop
085	R/S	51	
086	GTO9	22 09	
087	*LBLD	21 14	
088	2	02	--
089	4	04	Subroutine to correct for time greater
090	X≤Y?	16-35	than 24 hours
091	GSB8	23 08	
092	X⇄Y	-41	
093	EEX	-23	
094	2	02	
095	÷	-24	
096	RTN	24	
097	*LBL8	21 08	--
098	CHS	-22	Subroutine to correct time
099	HMS+	16-55	
100	SF2	16 21 02	
101	X⇄Y	-41	
102	RTN	24	
103	*LBLE	21 15	--
104	RCL0	36 00	Output subroutine
105	INT	16 34	Print onstation, offstation, and land
106	F2?	16 23 02	
107	GSB2	23 02	
108	+	-55	
109	PRTX	-14	
110	STO0	35 00	
111	RTN	24	
112	*LBL2	21 02	--
113	1	01	Subroutine to correct date
114	+	-55	
115	RTN	24	
116	*LBL0	21 00	--
117	FRC	16 44	Subroutine to change .hhmm to hh.mm
118	EEX	-23	
119	2	02	
120	×	-35	

121	RTN	24	Subroutine to correct for negative time
122	*LBLe	21 16 15	
123	3	03	
124	4	04	
125	HMS+	16-55	
126	SF3	16 21 03	
127	RTN	24	
128	*LBLd	21 16 14	
129	1	01	Subroutine to correct date
130	-	-45	
131	RTN	24	
132	*LBLa	21 16 11	Store crew postflight to preflight rest time
133	STO0	35 00	
134	CF2	16 22 02	
135	RTN	24	
136	*LBLA	21 11	
137	STOA	35 11	Compute crew's earliest possible takeoff
138	R↑	16-31	Store latest landing: ddd.hhmm
139	STOI	35 46	Store crew #
140	R↓	-31	
141	FRC	16 44	
142	EEX	-23	
143	2	02	
144	×	-35	
145	HMS+	16-55	
146	HMS+	16-55	
147	RCL0	36 00	
148	HMS+	16-55	Compute crew I's takeoff availability
149	GSBD	23 14	
150	STOi	35 45	
151	RCLA	36 11	
152	INT	16 34	
153	F2?	16 23 02	
154	GSB2	23 02	
155	ST+i	35-55 45	
156	RCLI	36 46	
157	DSP0	-63 00	
158	PRTX	-14	Print crew #
159	RCLi	36 45	
160	DSP4	-63 04	
161	PRTX	-14	Print crew's ddd.hhmm takeoff availability
162	SPC	16-11	
163	RTN	24	

164	*LBLE	21 15	List flight crews in order of availability
165	STOA	35 11	Store # of crews = N
166	0	00	Zero counter
167	STOC	35 13	
168	*LBL7	21 07	
169	RCLA	36 11	Begin loop
170	STOI	35 46	
171	STOB	35 12	
172	RCLi	36 45	Recall crew # N availability
173	STOD	35 14	
174	*LBLb	21 16 12	
175	DSZI	16 25 46	Recall crew # N-1 availability
176	GTO1	22 01	
177	GTO4	22 04	
178	*LBL1	21 01	
179	RCLD	36 14	Compare availability
180	RCLi	36 45	
181	X>Y?	16-34	
182	GSB3	23 03	
183	STOD	35 14	Store earliest ddd.hhmm
184	RCLI	36 46	
185	STOB	35 12	
186	GTOb	22 16 12	
187	*LBL3	21 03	
188	X⇄Y	-41	Swap registers
189	STOD	35 14	
190	GTOb	22 16 12	
191	*LBL4	21 04	
192	RCLB	36 12	Output routine
193	DSP0	-63 00	Print crew #
194	PRTX	-14	
195	STOI	35 46	
196	DSP4	-63 04	
197	RCLi	36 45	Print crew availability ddd.hhmm
198	PRTX	-14	
199	SPC	16-11	
200	SPC	16-11	
201	SPC	16-11	
202	SPC	16-11	
203	EEX	-23	
204	4	04	
205	ST+i	35-55 45	Add 1000 to day--i.e. create an artificially
206	1	01	large date
207	RCLC	36 13	
208	+	-55	
209	STOC	35 13	
210	RCLA	36 11	
211	X=Y?	16-33	Check for end of loop
212	GTO5	22 05	
213	GTO7	22 07	
214	*LBL5	21 05	
215	STOI	35 46	
216	*LBL6	21 06	
217	EEX	-23	
218	4	04	Subtract 10,000 from date--i.e. correct
219	ST-i	35-45 45	artificially
220	DSZI	16 25 46	
221	GTO6	22 06	
222	R/S	51	

VII. TARGET MOTION ANALYSIS (TMA) OF A BEARINGS-ONLY TARGET FROM A MOVING PLATFORM by LT P. W. Marzluff and LT R. C. Pilcher

A. Problem Statement

Bearings to a target either stationary or moving with constant course and speed are available from a non-stationary tracking platform. Determine the target's range, course and speed.

B. Operational Analysis

The four bearing TMA technique used in this program requires a minimum of four target bearing observations taken during a minimum of two tracking legs. A target bearing observation must be made and entered for the time corresponding to the initiation of own ship course or speed change. Exact target bearing observations of 090° and 270° require the addition of 0.1° to the observed value to avoid infinite computational values. When the tracking problem carries into a new day, the previous day's time scale must be continued (i.e. a time of 0010 on the second day must be enetred as 2410).

Own ship and target information is entered on card 1. Estimation of target parameters begins on card 1 and is completed on card 2. Entering supplemental target or own ship information and generating a new estimate again requires the use of card 1 and then card 2.

The accuracy of the estimates are dependent on the accuracy of the inputs, principally the target bearing, the magnitude

of course or speed change, and the number of observations made. For a more complete examination of the character of the estimates, see Reference 1.

C. Computational Algorithm

1. Input own ship's course and speed. Calculate and store velocity components.
2. Input time, t_i, and observed target bearing, B_i. Calculate elasped time since the first observation, Δt_i, and tan B_i. Calculate and store the matrix values.
3. When own ship changes course or speed enter t_i and B_i observed at the time the course or speed change was made. Enter new own ship course and speed prior to entering the next bearing observation.
4. When at least four target bearing observations have been entered (bearings taken on a minimum of two tracking legs), estimation of target range, course, and speed can be made.
5. Additional target and own ship information can be entered and new estimates made.

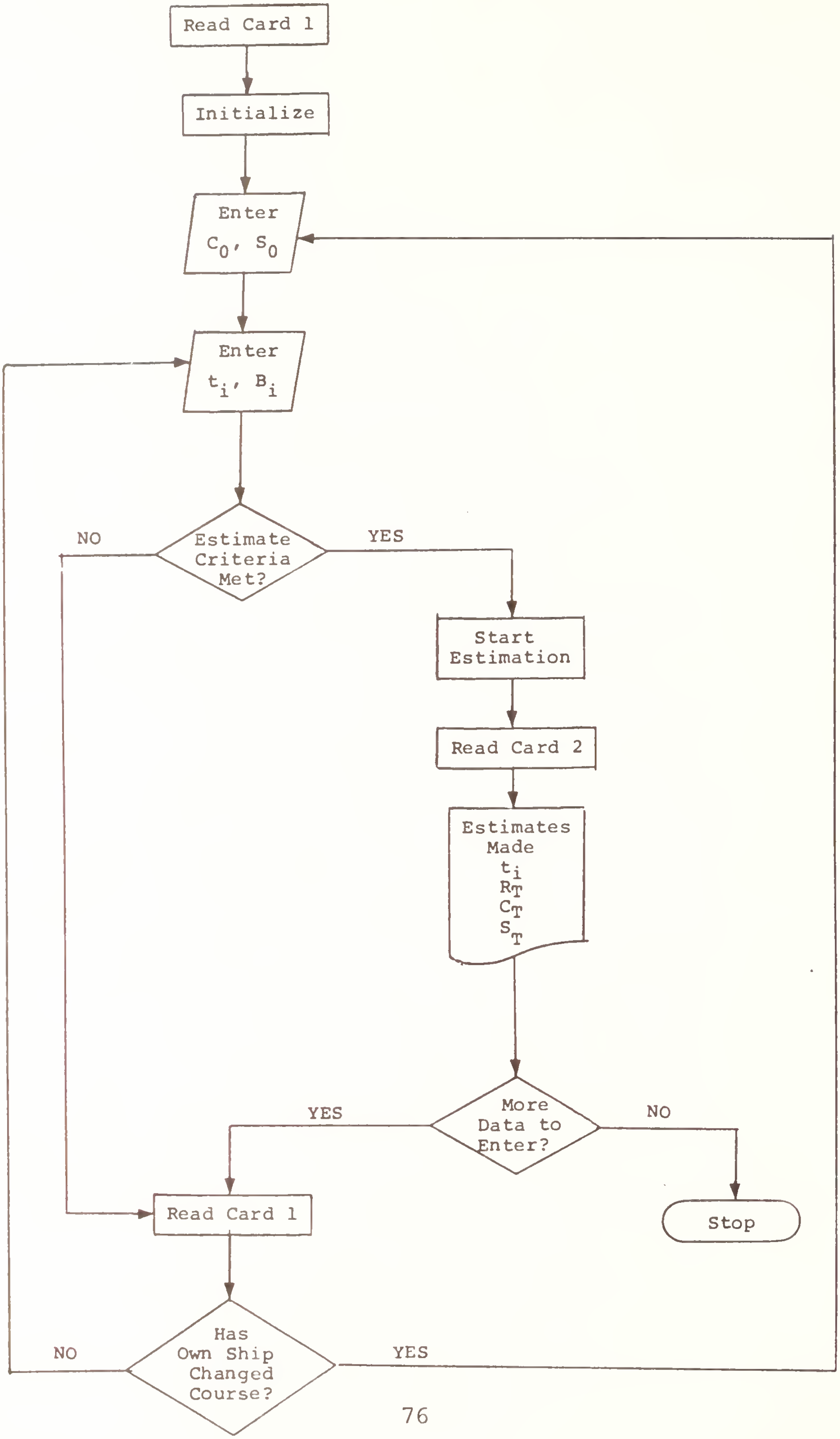
D. HP-67/97 Calculator Program

1. User Instructions

Step	Instruction	Input	Key(s)	Output
1.	Enter program card 1			
2.	Initialize		fe	0.00
3.	Enter own ship course and speed	C_0 (degrees) S_0 (knots)	↑ fa	 W
4.	Enter the time and the observed bearing	t_i (HH.MM) B_i (degrees)	↑ C	 i
5.	If estimation criteria is met go to Step 8; otherwise continue.			
6.	If own ship has changed course or speed go to Step 3; otherwise continue.			
7.	Go to Step 4.			
8.	Compute target range A. Start calculation B. Enter program Card 2 C. Continue calculation		 E A	 t_i; R_T (yds)
9.	Compute target course		C	C_T (degrees)
10.	Compute target speed		E	S_T (knots)
11.	If additional observations are to be entered, read program card 1 and go to Step 6.			

NOTE: ESTIMATION CRITERIA: A minimum of four bearing observations taken on a minimum of two own ship tracking legs must be entered prior to making estimates of target parameters.

User Instructions Flowchart



2. Sample Problems

a. Exact Bearing Information.

Own ship tracks on course 000° at 15 knots for 10 minutes and turns to 057° at 15 knots. Three bearing observations are made on the first leg, with the fourth observation made on the second leg. Exact bearing information is assumed available. The contact is tracked as follows:

Time	Bearing
1000	150.0
1005	148.2
1010	146.9
1020	154.7

Estimate the target's range, course and speed at time 1020. (Answers: 12,920 yds, 045°, and 10.0 knots.)

The following additional observations are made:

Time	Bearing
1025	158.8
1030	163.0
1035	167.2

Estimate the target's range, course and speed at time 1035. (Answers: 12540 yds, 045°, 10.0 knots.)

	GSBe	Initialize
	000.00 ENT↑ 15.00 GSBa	Own ship's course and speed
	10.00 ENT↑ 150.00 GSBC	Observation #1 time and bearing
CARD 1 card	10.05 ENT↑ 148.20 GSBC	Observation #2 time and bearing
	10.10 ENT↑ 146.90 GSBC	Observation #3 time and bearing
	057.00 ENT↑ 15.00 GSBa	New own ship course and speed
	10.20 ENT↑ 154.70 GSBC	Observation #4 time and bearing
	GSBE	Start estimation
CARD 2	GSBA 10.20 *** 12901.01 ***	Continue estimation Time of estimate Estimated target range in yards
	GSBC 48.37 ***	Estimated target course in degrees
	GSBE 9.86 ***	Estimated target speed in knots
	10.25 ENT↑ 158.80 GSBC	Observation #5 time and bearing
CARD 1	10.30 ENT↑ 163.00 GSBC	Observation #6 time and bearing
	10.35 ENT↑ 167.20 GSBC	Observation #7 time and bearing
	GSBE	Start estimation
CARD 2	GSBA 10.35 *** 12540.49 ***	Continue estimation Time of revised estimate Revised estimated target range
	GSBC 46.22 ***	Revised estimated target course
	GSBE 9.95 ***	Revised estimated target speed

b. Inaccurate Bearing Information

Own ship tracks on course 000° at 15 knots for 10 minutes and turns to 056° at 15 knots. Three bearing observations are made on the first leg, with the fourth observation made on the second leg. The true target bearings have been altered by normal random variable with mean zero and variance 0.5 degrees squared. The contact is tracked as follows:

Time	Bearing
1000	150.0
1005	148.7
1010	146.0
1020	155.2

Estimate the target's range, course and speed at time 1020. (Answers: 12,920 yds, 045°, and 10.0 knots.)

The following additional observations are made:

Time	Bearing
1025	159.7
1030	163.0
1035	166.4

Estimate the target's range, course and speed at time 1035. (Answers: 12,740 yds, 045°, and 10.0 knots.)

CARD 1	GSBe	Initialize
	000.00 ENT↑ 15.00 GSBa	Own ship's course and speed
	10.00 ENT↑ 150.00 GSBC	Observation #1 time and bearing
	10.05 ENT↑ 148.70 GSBC	Observation #2 time and bearing
	10.10 ENT↑ 146.00 GSBC	Observation #3 time and bearing
	056.00 ENT↑ 15.00 GSBa	New own ship course and speed
	10.20 ENT↑ 155.20 GSBC	Observation #4 time and bearing
	GSBE	Start estimation
CARD 2	GSBA 10.20 *** 5757.14 ***	Continue estimation Time of estimate Estimated target range in yards
	GSBC 338.26 ***	Estimated target course in degrees
	GSBE 83.67 ***	Estimated target speed in knots
CARD 1	10.25 ENT↑ 159.70 GSBC	Observation #5 time and bearing
	10.30 ENT↑ 163.00 GSBC	Observation #6 time and bearing
	10.35 ENT↑ 166.40 GSBC	Observation #7 time and bearing
	GSBE	Start estimation
CARD 2	GSBA 10.35 *** 12945.90 ***	Continue estimation Time of revised estimate Revised estimated target range
	GSBC 68.64 ***	Revised estimated target course
	GSBE 9.87 ***	Revised estimated target speed

3. Program Storage Allocation and Listing

Registers

R0:	t_1	S0:	PS1; PS3; PS10
R1:	$\Sigma W_i - \Sigma Z_i \tan B_i$	S1:	$\Sigma \Delta t_i \tan^2 B_i$
R2:	$-\Sigma W_i \tan B_i + \Sigma Z_i \tan^2 B_i$	S2:	$\Sigma \Delta t_i^2 \tan^2 B_i$
R3:	$\Sigma W_i \Delta t_i - \Sigma Z_i \Delta t_i \tan B_i$	S3:	$-\Sigma \Delta t_i^2 \tan B_i$
R4:	$-\Sigma W_i \Delta t_i \tan B_i + \Sigma Z_i \Delta t_i \tan^2 B_i$	S4:	$\Sigma \Delta t_i$
R5:	t_{i-1}	S5:	$\Sigma \Delta t_i^2$
R6:	t_i	S6:	$\Sigma \tan B_i$
R7:	W_i	S7:	$\Sigma \tan^2 B_i$
R8:	Z_i	S8:	$\Sigma \Delta t_i \tan B_i$
R9:	PS11; PS12; $\hat{v}$; Δt_i	S9:	$i = N$

RA: $\dot{W}$

RB: $\dot{Z}$

RC: Δt_i; PS4; $\hat{u}$

RD: $\tan B_i$; PS7; $\hat{v}$

RE: $Z_i \tan B_i$; PS13; $\hat{X}_1$

RI: i; PS2; PS5; PS6; PS8; PS9; $\hat{Y}_1$

NOTE: All summations are over the range of $i = 1,\ldots,N$.

PS denotes 'partial-sum'.

Initial Flag Status and Use:

0: OFF, Unused

1: OFF, Used

2: OFF, Unused

3: OFF, Used

User Control Keys; Card 1:

A:

B:

C: $t_i \uparrow B_i \rightarrow$

D:

E: Start $\rightarrow$

a: $C_0 \uparrow S_0 \rightarrow$

b:

c:

d:

e: Initialize $\rightarrow$

User Control Keys; Card 2:

A: $\rightarrow t_i; \hat{R}_{T_i}$

B:

C: $\rightarrow \hat{C}_T$

D:

E: $\rightarrow \hat{S}_{T_i}$

a:

b:

c:

d:

e:

Card 1

001	*LBL1	21 01	Stores initial time, t_1, for further use
002	STO0	35 00	
003	STO5	35 05	
004	RTN	24	
005	*LBLe	21 16 15	Initialization
006	CLRG	16-53	
007	P⇄S	16-51	
008	CLRG	16-53	
009	CLX	-51	
010	SF2	16 21 02	
011	RTN	24	
012	*LBLa	21 16 11	Calculates and stores own ship velocity components
013	→R	44	
014	STOB	35 12	
015	R↓	-31	
016	STOA	35 11	
017	RTN	24	
018	*LBLC	21 13	Input observation and bearing
019	TAN	43	
020	STOD	35 14	Fills the matrix values
021	X⇄Y	-41	
022	HMS→	16 36	
023	STO6	35 06	
024	F2?	16 23 02	
025	GSB1	23 01	Branches to store initial time
026	RCL0	36 00	
027	-	-45	
028	STOC	35 13	
029	Σ+	56	
030	STOI	35 46	
031	R↓	-31	
032	RCLC	36 13	
033	X²	53	
034	×	-35	
035	P⇄S	16-51	
036	ST-3	35-45 03	
037	RCLD	36 14	
038	×	-35	
039	ST+2	35-55 02	
040	RCLC	36 13	
041	RCLD	36 14	
042	X²	53	
043	×	-35	
044	ST+1	35-55 01	
045	P⇄S	16-51	
046	RCL6	36 06	
047	RCL5	36 05	
048	-	-45	
049	ENT↑	-21	
050	ENT↑	-21	

Card 1

051	RCLA	36 11	
052	x	-35	
053	ST+7	35-55 07	
054	R↓	-31	
055	RCLB	36 12	
056	x	-35	
057	ST+8	35-55 08	
058	RCL6	36 06	
059	STO5	35 05	
060	RCL7	36 07	
061	ST+1	35-55 01	
062	RCLD	36 14	
063	x	-35	
064	ST-2	35-45 02	
065	RCLC	36 13	
066	x	-35	
067	ST-4	35-45 04	
068	RCL7	36 07	
069	RCLC	36 13	
070	x	-35	
071	ST+3	35-55 03	
072	RCL8	36 08	
073	RCLD	36 14	
074	x	-35	
075	STOE	35 15	
076	ST-1	35-45 01	
077	RCLD	36 14	
078	x	-35	
079	ST+2	35-55 02	
080	RCLC	36 13	
081	x	-35	
082	ST+4	35-55 04	
083	RCLE	36 15	
084	RCLC	36 13	
085	x	-35	
086	ST-3	35-45 03	
087	RCLI	36 46	
088	RTN	24	
089	*LBLE	21 15	Start estimate calculations
090	P⇄S	16-51	
091	1	01	Changes the sign of two matrix elements
092	CHS	-22	
093	ST×6	35-35 06	
094	ST×8	35-35 08	
095	RCL8	36 08	
096	RCL7	36 07	(Partial sums stored throughout; PS)
097	x	-35	
098	RCL6	36 06	
099	RCL1	36 01	
100	x	-35	

Card 1

101	-	-45
102	RCL4	36 04
103	X^2	53
104	RCL9	36 09
105	RCL5	36 05
106	x	-35
107	-	-45
108	x	-35
109	STO0	35 00
110	RCL6	36 06
111	RCL4	36 04
112	x	-35
113	RCL9	36 09
114	RCL8	36 08
115	x	-35
116	-	-45
117	RCL8	36 08
118	X^2	53
119	RCL6	36 06
120	RCL3	36 03
121	x	-35
122	-	-45
123	x	-35
124	ST-0	35-45 00
125	RCL6	36 06
126	X^2	53
127	RCL9	36 09
128	RCL7	36 07
129	x	-35
130	-	-45
131	RCL8	36 08
132	P⇄S	16-51
133	RCL3	36 03
134	x	-35
135	RCL4	36 04
136	P⇄S	16-51
137	RCL4	36 04
138	x	-35
139	-	-45
140	x	-35
141	STOI	35 46
142	RCL8	36 08
143	X^2	53
144	RCL4	36 04
145	RCL1	36 01
146	x	-35
147	-	-45
148	RCL6	36 06
149	P⇄S	16-51
150	RCL1	36 01

151	×	-35
152	RCL2	36 02
153	P⇄S	16-51
154	RCL9	36 09
155	×	-35
156	-	-45
157	×	-35
158	RCLI	36 46
159	-	-45
160	STOC	35 13
161	RCL0	36 00
162	×	-35
163	P⇄S	16-51
164	STO9	35 09
165	P⇄S	16-51
166	RCL6	36 06
167	X²	53
168	RCL9	36 09
169	RCL7	36 07
170	×	-35
171	-	-45
172	RCL6	36 06
173	RCL5	36 05
174	×	-35
175	RCL4	36 04
176	RCL3	36 03
177	×	-35
178	-	-45
179	×	-35
180	STOI	35 46
181	RCL8	36 08
182	X²	53
183	RCL4	36 04
184	RCL1	36 01
185	×	-35
186	-	-45
187	RCL6	36 06
188	RCL4	36 04
189	×	-35
190	RCL9	36 09
191	RCL8	36 08
192	×	-35
193	-	-45
194	×	-35
195	RCLI	36 46
196	-	-45
197	STOD	35 14
198	RCL6	36 06
199	RCL4	36 04
200	×	-35

Card 1

201	RCL9	36 09
202	RCL8	36 08
203	×	-35
204	-	-45
205	RCL8	36 08
206	P⇄S	16-51
207	RCL2	36 02
208	×	-35
209	RCL4	36 04
210	P⇄S	16-51
211	RCL6	36 06
212	×	-35
213	-	-45
214	×	-35
215	STOI	35 46
216	RCL8	36 08
217	RCL7	36 07
218	×	-35
219	RCL6	36 06
220	RCL1	36 01
221	×	-35
222	-	-45
223	R/S	51

Card 2

001	*LBLA	21 11	Calculation of estimates
002	RCL4	36 04	
003	P⇄S	16-51	
004	RCL1	36 01	
005	×	-35	
006	RCL3	36 03	
007	P⇄S	16-51	
008	RCL9	36 09	
009	×	-35	
010	−	-45	
011	×	-35	
012	RCLI	36 46	
013	−	-45	
014	RCLD	36 14	
015	×	-35	
016	P⇄S	16-51	
017	RCL9	36 09	
018	X⇄Y	-41	
019	−	-45	
020	STO9	35 09	
021	P⇄S	16-51	
022	RCL6	36 06	
023	X²	53	
024	RCL9	36 09	
025	RCL7	36 07	
026	×	-35	
027	−	-45	
028	RCL8	36 08	
029	RCL3	36 03	
030	×	-35	
031	RCL4	36 04	
032	RCL2	36 02	
033	×	-35	
034	−	-45	
035	×	-35	
036	STOI	35 46	
037	RCL8	36 08	
038	X²	53	
039	RCL4	36 04	
040	RCL1	36 01	
041	×	-35	
042	−	-45	
043	RCL6	36 06	
044	RCL8	36 08	
045	×	-35	
046	RCL9	36 09	
047	RCL1	36 01	
048	×	-35	
049	−	-45	
050	×	-35	

Card 2

051	RCLI	36 46	
052	-	-45	
053	STOE	35 15	
054	RCL0	36 00	
055	x	-35	
056	STO0	35 00	
057	RCL6	36 06	
058	RCL4	36 04	
059	x	-35	
060	RCL9	36 09	
061	RCL5	36 05	
062	x	-35	
063	-	-45	
064	RCL5	36 05	
065	RCL1	36 01	
066	x	-35	
067	RCL6	36 06	
068	RCL2	36 02	
069	x	-35	
070	-	-45	
071	x	-35	
072	STOI	35 46	
073	RCL8	36 08	
074	RCL7	36 07	
075	x	-35	
076	RCL6	36 06	
077	RCL1	36 01	
078	x	-35	
079	-	-45	
080	RCL4	36 04	
081	RCL8	36 08	
082	x	-35	
083	RCL9	36 09	
084	RCL3	36 03	
085	x	-35	
086	-	-45	
087	x	-35	
088	RCLI	36 46	
089	-	-45	
090	RCLD	36 14	
091	x	-35	
092	RCL0	36 00	
093	X⇄Y	-41	
094	-	-45	
095	P⇄S	16-51	
096	RCL9	36 09	
097	X⇄Y	-41	
098	÷	-24	
099	STO9	35 09	^
100	RCLE	36 15	v

Card 2

101	×	-35	
102	RCLC	36 13	
103	X⇄Y	-41	
104	-	-45	
105	RCLD	36 14	
106	÷	-24	$\hat{u}$
107	STOC	35 13	
108	RCL9	36 09	
109	STOD	35 14	
110	P⇄S	16-51	
111	RCL6	36 06	
112	RCL8	36 08	
113	×	-35	
114	RCL9	36 09	
115	RCL1	36 01	
116	×	-35	
117	-	-45	
118	×	-35	
119	CHS	-22	
120	RCL6	36 06	
121	RCL4	36 04	
122	×	-35	
123	RCL9	36 09	
124	RCL8	36 08	
125	×	-35	
126	-	-45	
127	RCLC	36 13	
128	×	-35	
129	-	-45	
130	RCL6	36 06	
131	P⇄S	16-51	
132	RCL1	36 01	
133	×	-35	
134	RCL2	36 02	
135	P⇄S	16-51	
136	RCL9	36 09	
137	×	-35	
138	-	-45	
139	+	-55	
140	RCL6	36 06	
141	X²	53	
142	RCL9	36 09	
143	RCL7	36 07	
144	×	-35	
145	-	-45	
146	÷	-24	$\hat{Y}_1$
147	STOI	35 46	
148	P⇄S	16-51	
149	RCL1	36 01	
150	P⇄S	16-51	

151	RCL8	36 08	
152	RCLD	36 14	
153	×	-35	
154	-	-45	
155	RCL4	36 04	
156	RCLC	36 13	
157	×	-35	
158	-	-45	
159	RCL6	36 06	
160	RCLI	36 46	
161	×	-35	
162	-	-45	
163	RCL9	36 09	
164	÷	-24	$\hat{X}_1$
165	STOE	35 15	
166	1	01	Change sign of two matrix elements
167	CHS	-22	
168	ST×6	35-35 06	
169	ST×8	35-35 08	
170	P⇄S	16-51	
171	RCL6	36 06	
172	→HMS	16 35	t_i
173	PRTX	-14	
174	RCL6	36 06	
175	RCL0	36 00	
176	-	-45	
177	STO9	35 09	
178	RCLC	36 13	
179	×	-35	
180	RCLE	36 15	
181	+	-55	
182	RCL7	36 07	
183	-	-45	
184	X²	53	
185	RCL9	36 09	
186	RCLD	36 14	
187	×	-35	
188	RCLI	36 46	
189	+	-55	
190	RCL8	36 08	
191	-	-45	
192	X²	53	
193	+	-55	
194	√X	54	$\hat{R}_T = ((X_1 + u,\ t_i - W_i)^2 + (Y_1 + v,\ t_i - Z_i)^2)^{1/2}$
195	2	02	
196	EEX	-23	(in nautical miles)
197	3	03	
198	×	-35	$\hat{R}_T$ in yards
199	PRTX	-14	
200	SF1	16 21 01	

Card 2

201	RTN	24	
202	*LBLC	21 13	
203	RCLC	36 13	
204	RCLD	36 14	
205	→P	34	
206	X⇄Y	-41	
207	X<0?	16-45	
208	GSB2	23 02	
209	PRTX	-14	$\hat{C}_T$
210	CF1	16 22 01	
211	RTN	24	
212	*LBL2	21 02	
213	3	03	
214	6	06	
215	0	00	
216	+	-55	
217	RTN	24	
218	*LBLE	21 15	
219	F1?	16 23 01	
220	GTO9	22 09	
221	R↓	-31	
222	PRTX	-14	$\hat{S}_T$
223	SPC	16-11	
224	RTN	24	

E. Mathematical Analysis

a. Assumptions

All calculations use a rectangular coordinate system as defined below. Additionally it is assumed the following quantities are accurately known (although the accuracy of the observed bearing varies):

(1) time

(2) target bearings

(3) observer course and speed

(4) observer initial position.

b. Symbology

(1) Observer

(a) W: East-West position

(b) Z: North-South position

(c) (W_i, Z_i): position at ith observation.

(2) Target

(a) X: East-West position

(b) Y: North-South position

(c) (X_i, Y_i): position at ith observation

(d) u: East-West velocity component

(e) v: North-South velocity component.

(3) Other

(a) B_i: measured bearing from observer to target at the ith observation.

(b) t_i: time of the ith observation

(c) Δt_i: elapsed time between ith and first observation.

(d) $i = 1$ is the initial observation.

c. Development

The geometry for a two leg TMA is shown in Figure 1. The target motion/position at any time can be described in terms of its initial position (X_1, Y_1), and its velocity components (u and v), which are unknown and the elapsed time, Δt_i, which is known by: $X_i = X_1 + u\Delta t_i$ and $Y_i = Y_1 + v\Delta t_i$. Knowledge of the target bearing leads to the following:

$$\tan B_i = \frac{X_i - W_i}{Y_i - Z_i} = \frac{X_1 + u\Delta t_i - W_i}{Y_1 + v\Delta t_i - Z_i}$$

or

$$X_1 - Y_1 \tan B_i + u\Delta t_i - v\Delta t_i \tan B_i = W_i - Z_i \tan B_i \qquad (1)$$

Equation (1) has four known variables $(W_i, Z_i, \Delta t_i, B_i)$ and the four unknown target variables (X_1, Y_1, u, v). Define estimates for the four unknowns as $\hat{X}_1$, $\hat{Y}_1$, $\hat{u}$, and $\hat{v}$ and define an error, e_i, that represents the errors due to the use of estimates in Equation (1) at each observation:

$$e_i = \hat{X}_1 - \hat{Y}_1 \tan B_i + \hat{u}\Delta t_i - \hat{v}\Delta t_i \tan B_i - W_i + Z_i \tan B_i .$$

The least squares estimates of the variables X_1, Y_1, u and v are those values which minimize the expression

$$\sum_{i=1}^{n} e_i^2 = E(X_1, Y_1, u, v)$$

where n is the number of observations.

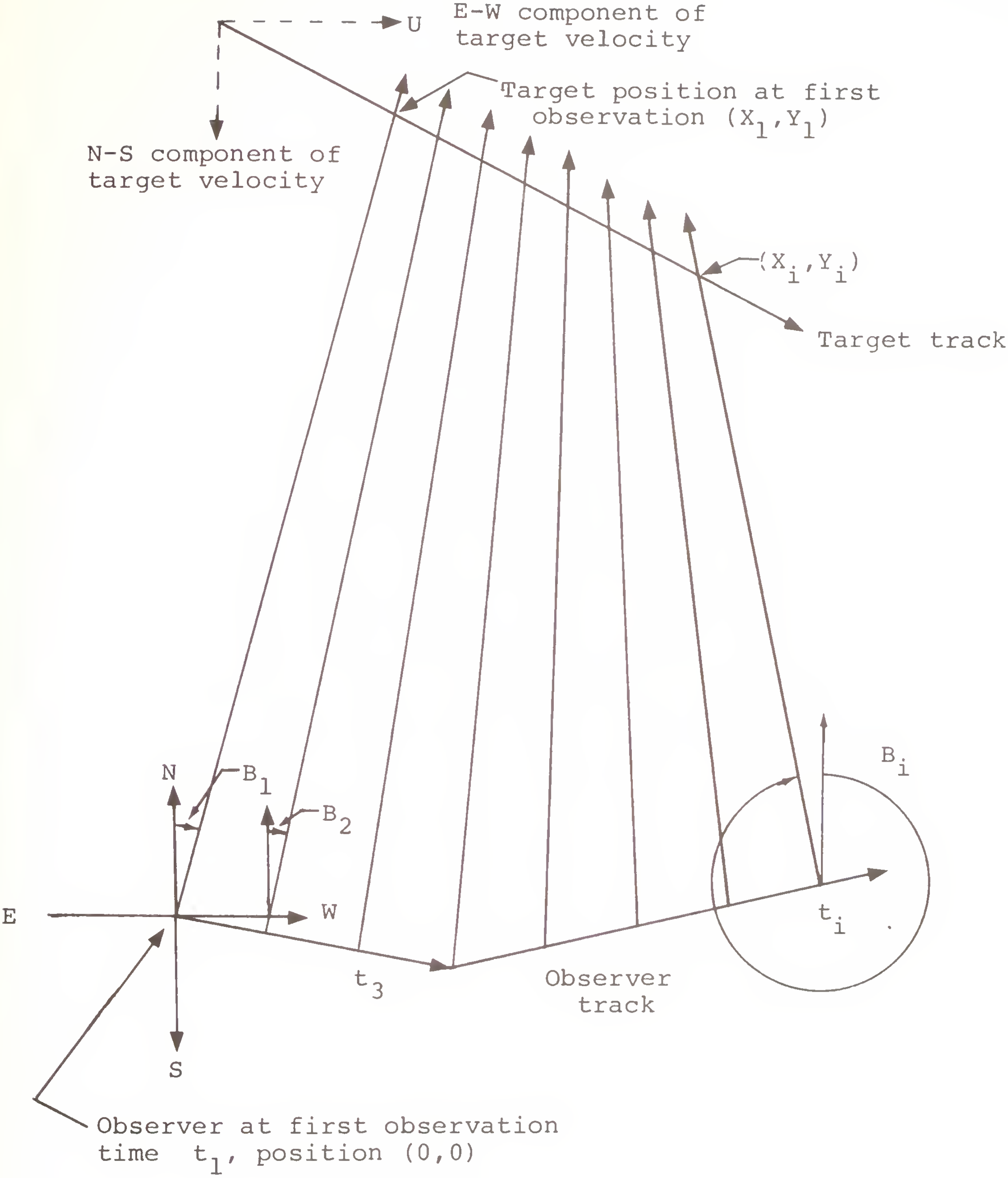


FIGURE 1. TMA Geometry

To minimize, take the partial derivatives of E with respect to each variable and set each partial derivative equal to zero. The resulting set of four linear equations in four unknowns (Figure 2) can be solved at each bearing observation if $i \geq 4$ and an observer course or speed change has occurred during the TMA period.

The following target parameters can then be calculated at each successive observation:

$$\text{Target Range} = \sqrt{(\hat{X}_i - W_i)^2 + (\hat{Y}_i - Z_i)^2}$$

$$\text{Target Speed} = \sqrt{\hat{u}^2 + \hat{v}^2}$$

Target Course = arctan$(\hat{u}/\hat{v})$ with appropriate logic to select the correct coordinate quadrant.

When calculating these parameters appropriate constants are required to insure proper units.

F. Reference.

1. P. W. Marzluff and R. C. Pilcher, "Basic Calculator Methods of Bearings-Only Target Motives Analyses for a Moving Sensor (U). Naval Postgraduate School Thesis, December 1978.

FOUR BEARING TMA NORMAL EQUATIONS

$$\begin{bmatrix} n & -\sum_{i=1}^{n} \tan B_i & \sum_{i=1}^{n} \Delta t_i & -\sum_{i=1}^{n} \Delta t_i \tan B_i \\ -\sum_{i=1}^{n} \tan B_i & \sum_{i=1}^{n} \tan^2 B_i & -\sum_{i=1}^{n} \Delta t_i \tan B_i & \sum_{i=1}^{n} \Delta t_i \tan^2 B_i \\ \sum_{i=1}^{n} \Delta t_i & -\sum_{i=1}^{n} \Delta t_i \tan B_i & \sum_{i=1}^{n} \Delta t_i^2 & -\sum_{i=1}^{n} \Delta t_i^2 \tan B_i \\ -\sum_{i=1}^{n} \Delta t_i \tan B_i & \sum_{i=1}^{n} \Delta t_i \tan^2 B_i & -\sum_{i=1}^{n} \Delta t_i^2 \tan B_i & \sum_{i=1}^{n} \Delta t_i^2 \tan^2 B_i \end{bmatrix} \begin{bmatrix} \hat{X}_1 \\ \hat{Y}_1 \\ \hat{u} \\ \hat{v} \end{bmatrix} \begin{bmatrix} \sum_{i=1}^{n} W_i - \sum_{i=1}^{n} Z_i \tan B_i \\ -\sum_{i=1}^{n} W_i \tan B_i + \sum_{i=1}^{n} Z_i \tan^2 B_i \\ \sum_{i=1}^{n} W_i \Delta t_i - \sum_{i=1}^{n} Z_i \Delta t_i \tan B_i \\ -\sum_{i=1}^{n} W_i \Delta t_i \tan B_i + \sum_{i=1}^{n} Z_i \Delta t_i \tan^2 B_i \end{bmatrix}$$

FIGURE 2

VIII. NAVAL GUNFIRE SUPPORT GRID SPOT CONVERSIONS, TRUE WIND, AND TIME OF FLIGHT/MAXIMUM ORDINATE COMPUTATIONS FOR 5-INCH/54 PROJECTILE by LT Keith P. Curtis

A. Problem Statement

The success of Naval Gunfire Support operations in the Combat Information Center (CIC) is a function of rapid information processing and relay. Specifically, substantial error can be introduced by inaccurate grid spot conversions and to a lesser degree by improper computations of true wind. Also, commencement of a fire mission can be delayed waiting for Time of Flight (TOF) and/or Maximum Ordinate (Max Ord) information.

The inherent error of rapid calculations can be minimized by the use of the handheld programmable calculator. This paper addresses the use of the Hewlett Packard HP-67 to perform grid-spot conversions; compute true wind, TOF, and Max Ord.

B. Operational Analysis

The objective of this program is to provide a one-card program to accommodate the following:

1. Correct for magnetic variance for a geographic area.
2. Accept the Observer Target Line (OTL) in either mils magnetic or degree magnetic.
3. Perform precise grid spot conversion.
4. Provide Time of Flight information.

5. Provide Maximum Ordinate information.

6. Compute true wind.

C. <u>Computational Algorithm</u>

1. Enter magnetic variance.

2. Enter OTL either in mils magnetic or degree magnetic.

3. Enter observer "spots": left-right, add-drop (in yards).

4. Convert spots to East-West, North-South.

5. Enter range of shot and compute TOF and max ord.

6. Enter own ship course and speed, and relative wind to compute true wind.

7. Repeat Steps 3 through 6 as necessary.

D. HP-67 Calculator Program

1. User Instructions

Step	Instruction	Input	Key(s)	Output
1.	Read program card (both sides)			
2.	Enter magnetic variation (+ for East, - for West). If no variation is entered, 0 is used.	mag.var.	f b	
3.	Enter Observer Target Line in either:			
a.	degrees magnetic, or		A	OTL °T
b.	mils magnetic		f a	OTL °T
4 a.	Enter Left/Right spot (- for left, + for right)	L/R	↑	
b.	Enter Add/Drop spot (- for drop, + for add)	A/D	B	
c.	Display E/W spot (+ E, - W)			E/W spot
d.	Display N/S spot (+ N, - S)			N/S spot
	Optional: Recover the E/W spot		h $x \leftrightarrows y$	E/W spot
e.	Repeat Step 4 as required until OTL changes			
5.	Compute TOF and max ord			
a.	Enter target range, display:	range	C	
b.	Time of Flight			TOF
c.	Max Ord			Max ord
	Optimal: Recover TOF		h R ↑	
6.	Compute true wind.			
a.	Enter own ship's course and speed in the form SS.CCC where SS is the speed in integer knots and CCC is a three-digit course	SS.CCC	↑	
b.	Enter relative wind	SS.CCC	D	
c.	True wind is displayed			SS.CCC

Note: Use of the keys $\boxed{C}$ or $\boxed{D}$ do not require an OTL input.

2. Sample Problem

Own ship:	035T 12KTS
Relative wind:	225R 6KTS
Magnetic variance:	7E
OTL:	1930 mils mag
Spot:	Left 250, Drop 150
Range:	9600 yds

West 27, North 290

TOF: 16 seconds

Max Ord: 1050ft

True Wind: 230T 17KTS

```
     7. GSBb
  1930. GSBa
   116.  ***
  -250. ENT↑
  -150. GSBE
   -27.  ***
   290.  ***

  9600. GSBC
    16.  ***
  1050.  ***

12.035 ENT↑
 6.225 GSBC
17.230  ***
```

3. Program Storage Allocation and Listings

Registers

R0:	OTL °T	S0:		RA:	$\log_{10}$range
R1:	TOF/ Max Ord Coeff (R1–R8)	S1:		RB:	Ship's heading
R2:		S2:		RC:	L-R spot
R3:		S3:		RD:	A-D spot
R4:		S4:	ΣX	RE:	
R5:		S5:		RI:	Control
R6:		S6:	ΣY		
R7:		S7:			
R8:		S8:			
R9:	mag.var.	S9:			

Initial Flag Status and Use

0:	OFF, Unused	2:	OFF, Unused
1:	OFF, Unused	3:	OFF, Unused

User Control Keys

A:	OTL (degrees)	a:	OTL (mils)
B:	L-R spot, A-D spot	b:	mag.var.
C:	Range	c:	
D:	O/S c/S rel wind	d:	
E:		e:	

```
0.000000000    0
0.777500000    1
2.166000000    2
-0.267100000   3
0.361700000    4
0.072100000    5
0.109300000    6
0.850500000    7
0.283100000    8
0.000000000    9
0.000000000    A
0.000000000    B
0.000000000    C
0.000000000    D
0.000000000    E
0.000000000    I
```

COEFFICIENTS FOR MAX ORD/TOF EQNS

PRESTORED DATA

```
001  *LBLb    21 16 12
002   DSP0      -63 00
003   STO9       35 09
004    R/S          51
```

STORE MAG VAR

```
005  *LBLa    21 16 11
006     .          -62
007     0           00
008     5           05
009     6           06
010     2           02
011     5           05
012     x          -35
```

CONVERT MILS TO DEGREES

```
013  *LBLA       21 11
014   DSP0      -63 00
015   RCL9       36 09
016     +          -55
017   X<0?       16-45
018   GSBe    23 16 15
019   STO0       35 00
020    R/S          51
```

CONVERT OTL TO TRUE BRG

```
021  *LBLB       21 12
022   DSP0      -63 00
023    X⇄Y         -41
024    →P           34
025    X⇄Y         -41
026   RCL0       36 00
027     -          -45
028    X⇄Y         -41
029    →R           44
030   PRTX         -14
031    X⇄Y         -41
032    R/S          51
```

CONVERT OBSERVER GRID SPOTS TO E-W, N-S SPOTS

```
033  *LBLC       21 13
034   DSP0      -63 00
035    EEX         -23
036     3           03
037     ÷          -24
038    LOG       16 32
039   STOA       35 11
040   RCL6       36 06
041   RCL7       36 07
042   RCL8       36 08
043   RCLA       36 11
044     x          -35
045     +          -55
046   RCLA       36 11
047     x          -35
048     +          -55
049    10^x      16 33
```

COMPUTE TIME OF FLIGHT

```
050   PRTX         -14
051     4           04
052   STOI       35 46
053   RCL5       36 05
054  *LBLc    21 16 13
055   RCLA       36 11
056     x          -35
057   RCLi       36 45
058     +          -55
059   DSZI    16 25 46
060   GTOc    22 16 13
061    10^x      16 33
062    R/S          51
```

COMPUTER MAX ORDINATE

```
063  *LBLD       21 14
064   DSP0      -63 00
065    P⇄S       16-51
066   CLRG       16-53
067    P⇄S       16-51
068    X⇄Y         -41
069   GSBd    23 16 14
070    Σ-        16 56
071    R↑        16-31
072   RCLB       36 12
073     +          -55
074   GSBd    23 16 14
075    Σ+           56
076   RCLΣ       36 56
077    →P           34
078    RND       16 24
079    INT       16 34
080    X⇄Y         -41
081   X<0?       16-45
082   GSBe    23 16 15
083    EEX         -23
084     3           03
085     ÷          -24
086     +          -55
087   DSP3      -63 03
088    R/S          51
```

COMPUTE TRUE WIND

```
089  *LBLd  21 16 14
090    INT     16 34
091   LSTX     16-63
092    FRC     16 44
093   STOB     35 12
094    EEX       -23
095      3        03
096      x       -35
097    X⇄Y       -41
098     →R        44
099    RTN        24
100  *LBLe  21 16 15
101      3        03
102      6        06
103      0        00
104      +       -55
105    RTN        24
106    R/S        51
```

E. Geometric/Mathematical Analysis

1. Grid-Spot Conversion

The conversion of grid spots oriented to an Observer Target Line to an East-West, North-South orientation for input to a shipboard GFCS can be accomplished by a rotation of the OTL counterclockwise to 000 degree True after the OTL has been corrected to true bearing (Figure 1).

The rotation formulas are

$$\begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} x' \\ y' \end{bmatrix},$$

or

$$x' = x\cos\theta + y\sin\theta \quad \text{and} \quad y' = -x\sin\theta + y\cos\theta,$$

where

θ = OTL °T

x = L/R Spot,

y = A/D Spot,

x' = E-W Spot,

y' = N-S Spot

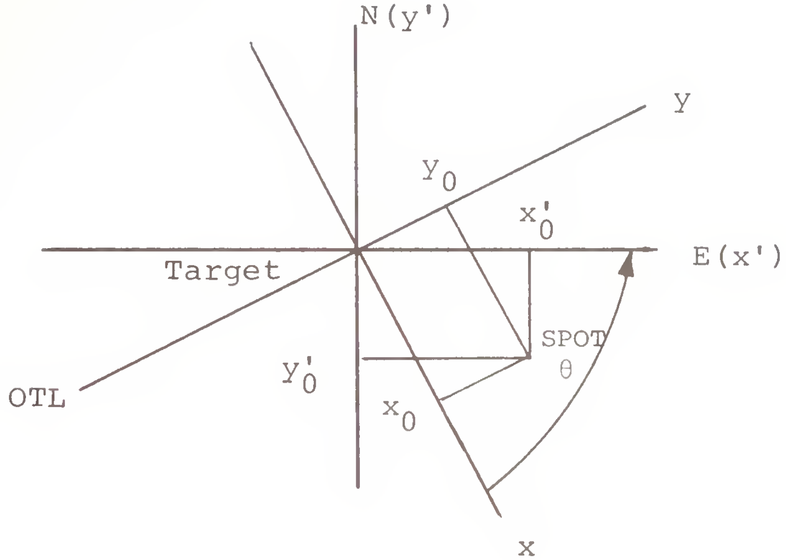


FIGURE 1: Rotation Geometry

2. <u>TOF and Maximum Ordinate</u>

This information is tabulated for the 5"/54 Projectile in BuOrd Publication OP1182 (Range Table for 5"/54). To find an equation which would best fit the tabulated data, a log transformation was made followed by a parabolic curve fit.

The time of Flight Equation is

$$f(x) = 10^{(0.1083 + 0.8505 \log x + 0.2831(\log x)^2)}$$

This equation generates solutions within 1 second of the tabulated values for ranges under 22,000 yds. The same approach was made in determining an equation for maximum ordinate.

The Maximum Ordinate Equation is

$$f(x) = 10^{(0.7775 + 2.1666 \log x - 0.2071(\log x)^2 + 0.2617(\log x)^3 + 0.0721(\log x)^4)}$$

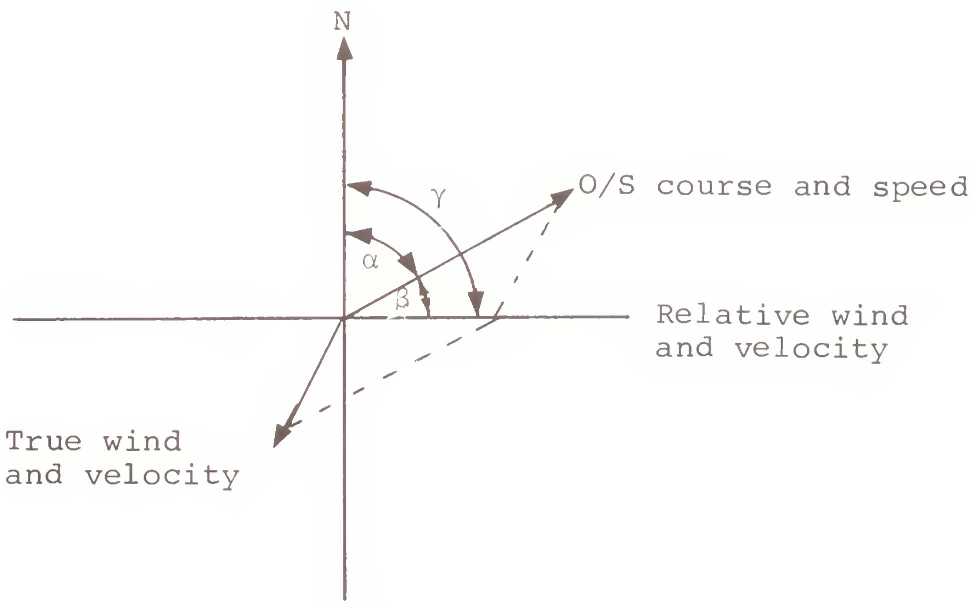
This equation generates solutions within 55ft of the tabulated values for ranges less than 19,000 yds. (Within 10ft for range less than 13,000 yds, and with 1 ft for ranges less than 7000 yds.)

The algorithm uses a nested polynomial to preserve accuracy in the calculation of the exponent. For example, Time of Flight is computed as follows:

$$f(x) = 10^{((0.2831 \log x + 0.8505)\log x + 0.1083)}$$

3. <u>True Wind</u>.

This computation is simple vector arithmetic. Relative wind is converted to Apparent wind by adding the ship's heading to the relative bearing, then converted to rectangular coordinates. Own ship's course and speed are then converted to rectangular coordinates and subtracted from apparent wind vector. The result is true wind which is converted to polar coordinates (Figure 2).



α = ship's heading

β = relative wind bearing

γ = apparent wind bearing

FIGURE 2. Wind vectors

IX. NORMAL MODE THEORY by LT J. M. Stone

A. Problem Statement

This program determines the number of normal modes that will propagate in a given ocean model. The ocean model must have either a rigid bottom or pressure release bottom. Also provided with each mode is the cutoff frequency (f_c), group velocity (C_g), and phase velocity (C_p). The user provides the speed of sound in water in m/sec (C_0), water depth in m(d), and frequency of the source in Hz(f).

B. Operational Analysis

None.

C. Computational Algorithm

Not submitted.

D. HP-67/07 Calculator Program

1. User Instructions

Step	Instruction	Input	Key(s)	Output
1.	Read program card			
2.	Initialize		f e	1.00
3.a.	Sound velocity (m/sec)	C_0	↑	
b.	Water depth (m)	d	↑	
c.	Source frequency (Hz)	f		
4.	Bottom type: Either			
a.	Rigid Bottom, or	none	A	
b.	Pressure release bottom		B	
5.	Output sequence			
a.	Mode number			n
b.	Cutoff frequency for mode n			f_c
c.	Group velocity for mode n			C_g
d.	Phase velocity for mode n			C_p
e.	Display mode number			n
6.	Continue from Step 4a or 4b depending upon original bottom type. This process continues until the highest mode that will propagate for the given conditions has been displayed when $\boxed{A}$ or $\boxed{B}$ is pressed and the next mode will not propagate then the program displays the mode number of the last mode that will propagate.			

2. Sample Problems

Example 1.

C_0 = 1500 m/sec　　　　Rigid bottom

d = 15 m

f = 150 Hz

GSBe	Initializes Program
1500.00 ENT↑	
15.00 ENT↑	
150.00 GSBA	
1.00 ***	Flashes mode number (n)
25.00 ***	Flashes f_c for n = 1
1479.02 ***	Flashes C_g for n = 1
1521.28 ***	Flashes C_p for n = 1 (Stops, displaying mode no.)
GSBA	
2.00 ***	Flashes n
75.00 ***	Flashes f_c for n = 2
1299.04 ***	Flashes C_g for n = 2
1732.05 ***	Flashes C_p for n = 2 (Stops, displaying mode no.)
GSBA	
3.00 ***	Flashes n
125.00 ***	Flashes f_c for n = 3
829.16 ***	Flashes C_g for n = 3
2713.68 ***	Flashes C_p for n = 3 (Stops, displaying mode no.)
GSBA	
3.00 ***	Mode 4 will not propagate under these conditions so regardless of how many times $\boxed{A}$ is pressed, mode 3 is displayed.
GSBA	
3.00 ***	

Example 2:

C_0 = 1500 m/sec Pressure Release Bottom

d = 15 m

f = 150 Hz

GSBe	Initializes Program
1500.00 ENT↑	
15.00 ENT↑	
150.00 GSBE	
1.00 ***	Flashes mode no.
50.00 ***	Flashes f_C for mode 1
1414.21 ***	Flashes C_g for mode 1
1590.99 ***	Flashes C_p for mode 1 (Stops, displaying mode no.)
GSBE	
2.00 ***	Flashes mode no.
100.00 ***	Flashes f_C for mode 2
1118.03 ***	Flashes C_g for mode 2
2012.46 ***	Flashes C_p for mode 2 (Stops, displaying mode no.)
GSBE	
2.00 ***	Only two modes will propagate
GSBE	
2.00 ***	

NOTE: If such conditions exist such that no modes will propagate, 0.00 is displayed.

The program stops between modes and requires the user to initiate the next mode in order to allow the user sufficient time to write down the information presented.

3. Program Storage Allocations and Program Listing

Registers:

R0; f	S0:	RA:
R1: d	S1:	RB:
R2: C_0	S2:	RC:
R3: n	S3:	RD:
R4: f_C	S4:	RE:
R5: $\sqrt{1 - (f_C/f)^2}$	S5:	RI:
R6:	S6:	
R7:	S7:	
R8:	S8:	
R9:	S9:	

Initial Flag Status and Uses

0: OFF, Unused	2. OFF, Unused
1: OFF, Unused	3: OFF, Unused

User Control Keys

A: Rigid Bottom	a:
B: Pressure Release Bottom	b:
C:	c:
D:	d:
E:	e:

001	*LBLe	21 16 15	Initializes Program
002	1	01	
003	STO3	35 03	n = 1
004	SF2	16 21 02	Controls storage of inputs on first pass
005	R/S	51	Input parameters at this stop
006	*LBLA	21 11	Case I--Rigid Bottom
007	F2?	16 23 02	
008	GSB1	23 01	Stores inputs on first pass
009	RCL3	36 03	
010	.	-62	
011	5	05	
012	x	-35	
013	.	-62	
014	2	02	Calculates f_C
015	5	05	
016	-	-45	$f_C = \frac{C_0(2n-1)}{4d}$
017	RCL2	36 02	
018	x	-35	
019	RCL1	36 01	
020	÷	-24	
021	RCL0	36 00	
022	X⇄Y	-41	
023	X≠Y?	16-32	Is $f_C \geq f$
024	X>Y?	16-34	
025	GTO4	22 04	Yes--display last mode # and stop
026	STO4	35 04	No--continue
027	RCL3	36 03	
028	PRTX	-14	Flash new mode #
029	RCL4	36 04	
030	PRTX	-14	Flash f_C
031	GSB2	23 02	Computes C_g and C_p
032	*LBL3	21 03	Increments mode no. for next iteration
033	1	01	
034	ST+3	35-55 03	
035	RCL3	36 03	
036	1	01	
037	-	-45	
038	R/S	51	Stops, displays mode # of run just completed
039	*LBLB	21 12	Case II--Pressure Release Bottom
040	F2?	16 23 02	
041	GSB1	23 01	Stores inputs on first pass
042	RCL3	36 03	
043	RCL2	36 02	
044	x	-35	Calculates f_C
045	RCL1	36 01	
046	÷	-24	
047	2	02	
048	÷	-24	$f_C = \frac{C_0(2n)}{4d}$
049	RCL0	36 00	

050	X⇄Y	-41	
051	X≠Y?	16-32	Is $f_C \geq f$
052	X>Y?	16-34	Display last mode no.
053	GTO4	22 04	Yes--and stop
054	STO4	35 04	No--continue
055	RCL3	36 03	
056	PRTX	-14	Flash new mode no.
057	RCL4	36 04	
058	PRTX	-14	Flash f_C
059	GSB2	23 02	Computes C_g and C_p increments mode # for next
060	GTO3	22 03	iteration
061	*LBL1	21 01	Subroutine 1--Stores inputs on first pass only
062	STO0	35 00	
063	R↓	-31	
064	STO1	35 01	
065	R↓	-31	
066	STO2	35 02	
067	RTN	24	
068	*LBL2	21 02	Subrounte 2--Calculates C_g and C_p
069	RCL4	36 04	
070	RCL0	36 00	
071	÷	-24	
072	X²	53	
073	CHS	-22	
074	1	01	$C_g = C_0 \sqrt{1 - (f_C/f)^2}$
075	+	-55	
076	√X	54	
077	STO5	35 05	
078	RCL2	36 02	
079	×	-35	
080	PRTX	-14	Flash C_g
081	RCL2	36 02	
082	RCL5	36 05	$C_p = C_0 / \sqrt{1 - (f_C/f)^2}$
083	÷	-24	
084	PRTX	-14	Flash C_p
085	RTN	24	
086	*LBL4	21 04	Allows the last mode number to be displayed
087	RCL3	36 03	after each iteration
088	1	01	
089	-	-45	
090	R/S	51	

E. Mathematical Analysis

Consider a shallow water ocean model with depth d. According to Normal Mode Theory the sound pressure at any point can be determined by the solution of the wave equation and is given by

$$p_n = A_n \sin K_{nz} z \exp[i(\omega t - K_{nz} x)]$$

Sound will propagate at different angles and this is what gives rise to the various modes. As θ, in Figure 1 approaches zero, the sound will not propagate because it is merely bouncing up and down off the surface and bottom (no x-direction of travel). This determines the cutoff frequency for mode n.

The cutoff frequency is dependent upon the boundary conditions at the surface and bottom because the solution of the wave equation is dependent upon the boundary conditions. The surface is always considered to be a pressure release boundary. As the mode number increases the cutoff frequency for that mode is higher also. When the cutoff frequency exceeds the frequency of the source then that mode will not propagate, nor will higher modes.

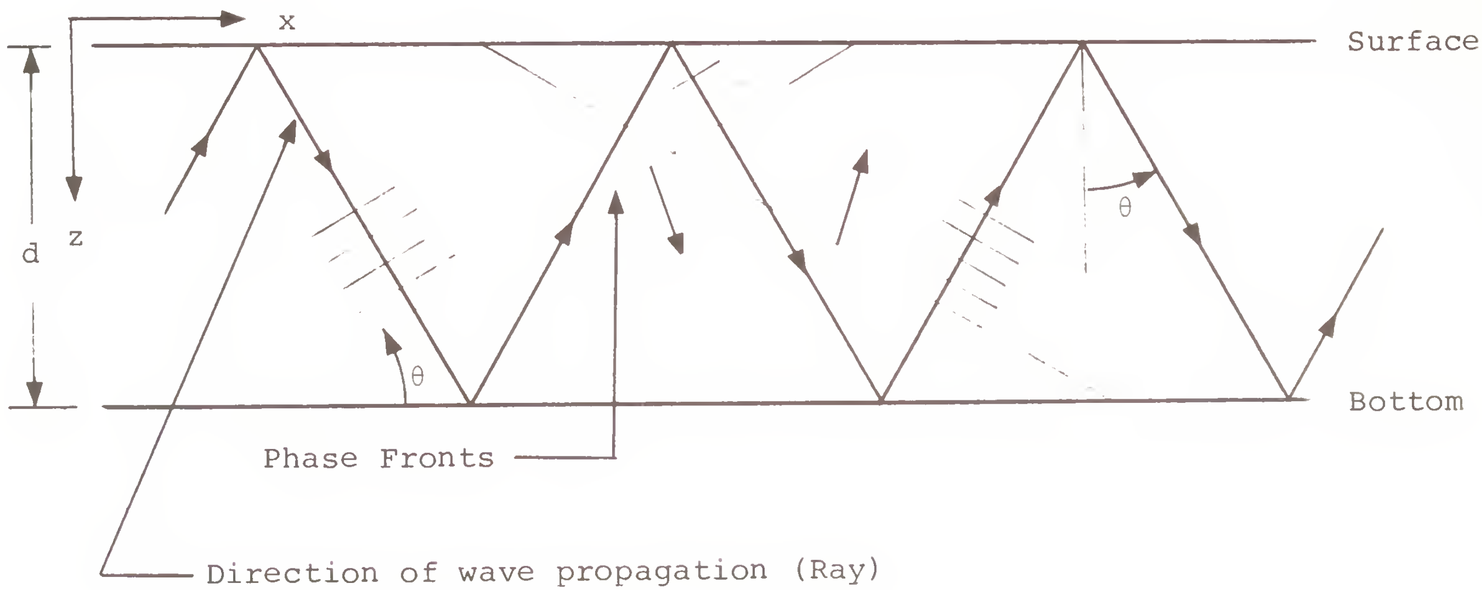


FIGURE 1. Wave Propagation Geometry

Case I: Rigid Bottom

For the rigid bottom, the boundary condition yields a

$$K_{nz} = \frac{(2n-1)\pi}{2d}, \qquad n = 1,2,3 \quad \text{(mode \#)}$$

and

$$f_C = \frac{C_0}{2} \frac{(2n-1)\pi}{2d} .$$

Case II: Pressure Release Bottom

For the pressure release bottom, the boundary condition yields

$$K_{nz} = \frac{n\pi}{d}, \qquad n = 1,2,3,... \quad \text{(mode \#)}$$

and

$$f_C = \frac{C_0}{2\pi} \frac{n\pi}{d} .$$

The remaining values calculated are the group velocity $C_g = C_0 \cos\phi$ and the phase velocity $C_p = C_0/\cos\phi$, where $\cos\phi = [1 - (f_C/f)^2]^{1/2}$, f_C is the cutoff frequency, f is the source frequency, and C_0 is the velocity of sound. The geometry of these quantities is shown in Figure 2.

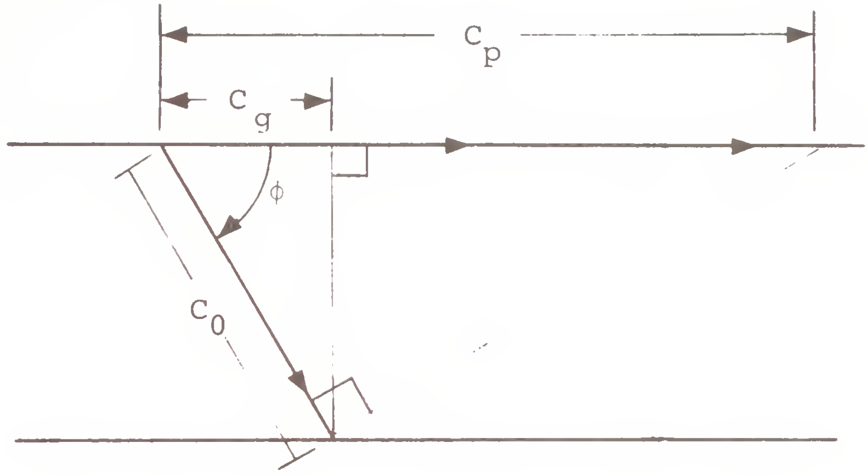
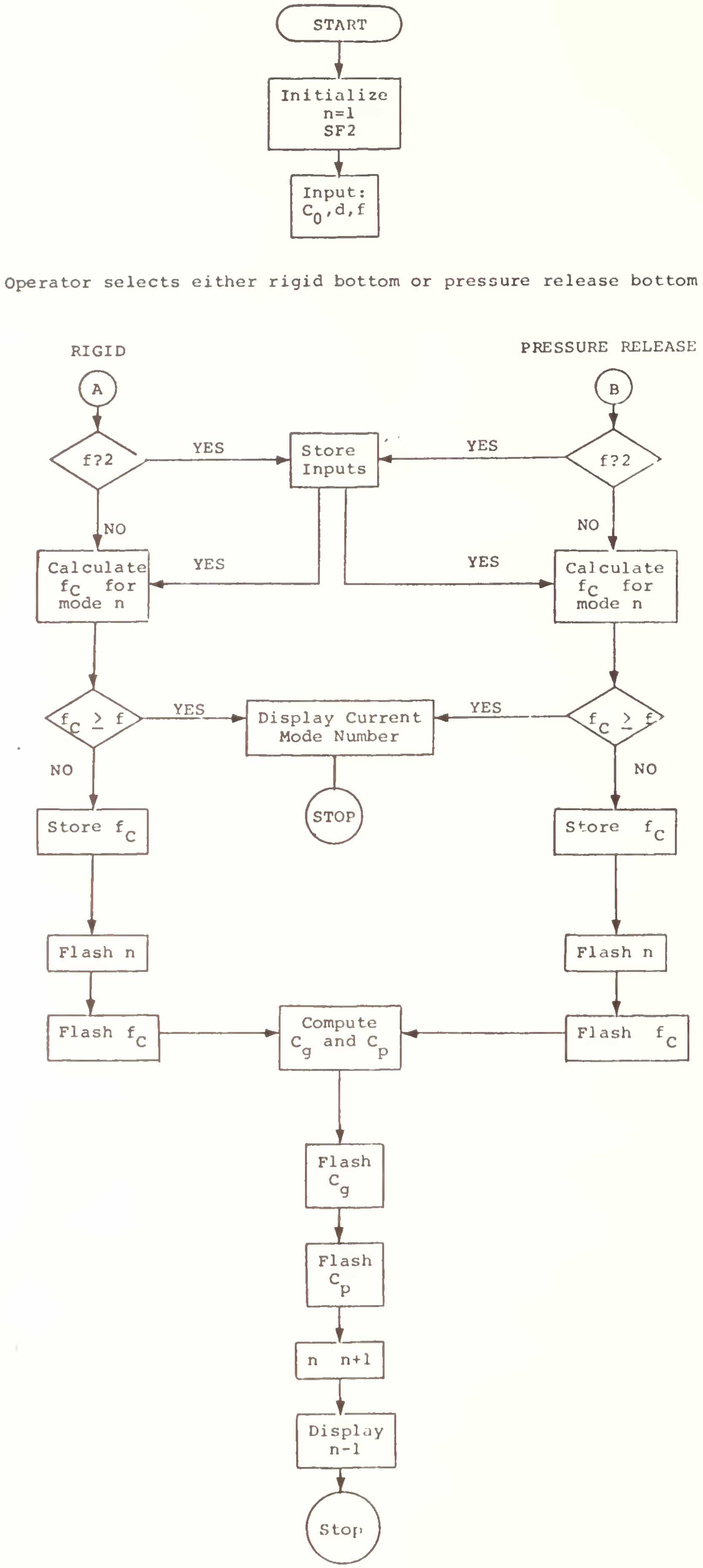


FIGURE 2. Group and Phase Velocity Geometry


START
Initialize
n=1
SF2
Input:
C_0,d,f
Operator selects either rigid bottom or pressure release bottom
RIGID
A
PRESSURE RELEASE
B
f?2
YES
Store
Inputs
YES
f?2
NO
NO
YES
YES
Calculate
f_C for
mode n
Calculate
f_C for
mode n
$f_C \geq f$
YES
Display Current
Mode Number
YES
$f_C \geq f$
NO
NO
STOP
Store f_C
Store f_C
Flash n
Flash n
Flash f_C
Compute
C_g and C_p
Flash f_C
Flash
C_g
Flash
C_p
n n+1
Display
n-1
Stop

X. NORMAL MODE TRANSMISSION LOSSES by LT Michael D. Clary

A. Problem Statement

As an alternative to Ray Theory, the use of Normal Mode Theory provides a more exact approach to the solution of transmission loss problems. Given source frequency and depth, range to and depth of receiver, and modal values for the speed of sound and the absorption coefficient, the program user may either specify the effective pressure amplitude of the source at one meter and solve for modal effective pressure amplitudes and the resulting transmission losses, or if the modal pressures are input, the value of the one meter effective source pressure amplitude can be obtained.

B. Operational Analysis

Since this program is used primarily as a theoretical problem solver based on give data, no operational analysis is provided.

C. Computational Algorithm HP-67

1. Input source frequency, depth, receiver depth, effective pressure amplitude at 1 meter (source) <u>or</u> effective pressure amplitude of given model.
2. Input absorption coefficient for given mode, range between source and receiver, and speed of sound for the mode.
3. Compute sums required for transmission loss equation.
4. Compute and output coherent and/or incoherent transmission losses.

D. HP-67 Calculator Program

1. User Instructions

Step	Instruction	Input	Keys(s)	Output
1.	Load side one and two of program card			
2.	Program the eigenfunction for $z_n(z)$, $z_n(z_0)$ at LBL3 in the W/PGRM MODE. (Assume value of z and z_0 will be in the x-register upon initialization of each computation.) The last two program steps <u>must</u> be STO(i), h RTN. Return calculator to RUN MODE			
3.	Enter computational values			
a.	frequency of source in Hz	f	ENTER	f
b.	depth of source in meters	z_0	ENTER	z_0
c.	depth of receiver in meters	z	ENTER	z
d.	effective pressure amplitude at 1 m in μPa	P(1)	A	f
	or effective pressure amplitude of given value	$-\|P_n\|$	A	f
	enter as $-\|P_n\|$			
e.	absorption coefficient for the mode	α_n	ENTER	α_n
f.	range from source in meters	r	ENTER	
g.	speed of sound for the mode in meters/sec	C_n	B	$\|P_n\|$ or P(1)
	Output will be either the mode value for the effective pressure amplitude or the value of the effective pressure amplitude at 1 m. All values are in μPa.			

Step	Instruction	Input	Key(s)	Output
4.	Sum the values of $P_n \sin(2\pi f_r)/C_n$ and $P_n \cos(2\pi f_r)/C_n$		C	n
5.	Sum the value of P_n^2		D	P_n^2
6.	Return to Step 2 for additional mode computations. Information for Steps 3a, b,c and d need not be re-entered. If all mode computations are completed, go to 7.			
7.	Compute transmission loss (in dB) for coherence for incoherence	 f	 E e	 T.L. T.L.
8.	For new case go to Step 1			

2. Sample Problem

For a nearly isovelocity layer of water 50 m deep overlying a silt bottom rich in decaying organic matter, the following are found to be good approximate values for a source frequency of 50 Hz:

n	$Z_n(z)$	c_n	f_n(cutoff)	α_n
1	0.2 sin(0.050z)	1550m/s	25 Hz	1×10^{-4}/m
2	0.2 sin(0.105z)	1630	33	2×10^{-4}
3	0.2 sin(0.189z)	2110	45	4×10^{-4}
4	not excited	---	55	---

a. Evaluate the effective amplitudes $|P_n|$ at a receiver at a range of 2 km and at a depth of 20 m. Determine both transmission loss values.

SOLUTION: (given that the source is at a depth of 50m)

1. Enter first eigenfunction $Z_n(z)$ under label 3. Value of z (or z_0) will be in the X-register.
2. Enter f, z_0, z and $P(1) = 10^7$ μPa for $n = 1$ and compute P_1.
3. After recording value P_1, sum the values for transmission loss calculations.
4. Repeat Steps 1-3 for all necessary values of n (three for the given example).
5. When all P_n's have been obtained, calculate transmission loss, both coherent and incoherent cases.
6. Numerical answers:

$P_1 = 2.05 \times 10^4$ μPa	$\|P_1\| = 2.05 \times 10^4$ μPa
$P_2 = -2.54 \times 10^4$ μPa	$\|P_2\| = 2.54 \times 10^4$ μPa
$P_3 = 0.39 \times 10^3$ μPa	$\|P_3\| = 0.39 \times 10^4$ μPa

TL(coherence) = 53 dB

TL(incoherence) = 50 dB

Keystroke Sequence for Sample Problem a.

```
                  GTO3
092   *LBL3      21 03
093       .        -62
094       0         00
095       5         05
096       x        -35
097     SIN         41
098       .        -62
099       2         02
100       x        -35
101    STOi      35 45
102     RTN         24
          50.00 ENT↑
          50.00 ENT↑
          20.00 ENT↑
          1.+07 GSBA
          50.00  ***
          1.-04 ENT↑
          2.+03 ENT↑
        1550.00 GSBB
        2.05+04  ***
                GSBC
        1.00+00  ***
                GSBD
        4.22+08  ***
                GTO3
092   *LBL3      21 03
093       .        -62
094       1         01
095       0         00
096       5         05
097       x        -35
098     SIN         41
099       .        -62
100       2         02
101       x        -35
102    STOi      35 45
103     RTN         24
```

```
          2.-04 ENT↑
          2.+03 ENT↑
          1630. GSBB
        2.54+04  ***
                GSBC
        2.00+00  ***
                GSBD
        6.44+02  ***
                GTO3
092   *LBL3      21 03
093       .        -62
094       1         01
095       8         08
096       9         09
097       x        -35
098     SIN         41
099       .        -62
100       2         02
101       x        -35
102    STOi      35 45
103     RTN         24
          4.-04 ENT↑
          2.+03 ENT↑
          2110. GSBB
        3.92+02  ***
                GSBC
        3.00+00  ***
                GSBD
        1.54+05  ***
                GSBE
            53.  ***
                GSBe
            50.  ***
```

b. Using the same table values and the computed values for each P_n, verify that the value for $P(1)$ is in fact 10^7 μPa.

1. Enter first eigenfunction $Z_n(z)$ under label 3.
2. Enter computational values for $n = 1$ and compute $P(1)$. Note that the values of P_n must be entered as the negative value of the absolute.
3. Calculate $P(1)$. Repeat Steps 1-3 for each value of n.
4. Numerical answers:

$|P_1| = 2.05 \times 10^4$ μPa $\qquad P(1) = 9.98 \times 10^6 \cong 10^7$ μPa

$|P_2| = 2.54 \times 10^4$ μPa $\qquad P(1) = 10^7$ μPa

$|P_3| = 0.39 \times 10^3$ μPa $\qquad P(1) = 9.94 \times 10^6 \cong 10^7$ μPa

Keystroke Sequence for Sample Problem b.

```
            GTO3
   50.00    ENT↑
   50.00    ENT↑
   20.00    ENT↑
-2.05+04    GSBA
   1.-04    ENT↑
   2.+03    ENT↑
 1550.00    GSBE
 9.98+06    ***
            GTO3
     50.    ENT↑
     50.    ENT↑
     20.    ENT↑
-2.54+04    GSBA
   2.-04    ENT↑
   2.+03    ENT↑
   1630.    GSBE
 1.00+07    ***
            GTO3
     50.    ENT↑
     50.    ENT↑
     20.    ENT↑
-0.39+03    GSBA
   4.-04    ENT↑
   2.+03    ENT↑
   2110.    GSBE
 9.94+06    ***
```

3. Program Storage Allocation and Program Listing

Registers:

R0:	f	S0:	ΣP_n^2
R1:	z_0	S1:	
R2:	z	S2:	
R3:	C_n	S3:	
R4:	α_n	S4:	$\Sigma \sin(2\pi f_r)/C_n$
R5:	r	S5:	
R6:	$P(1)$ or $-\lvert P_n\rvert$	S6:	$\Sigma \cos(2\pi f_r)/C_n$
R7:		S7:	$\Sigma \cos^2(2\pi f_r)/C_n$
R8:		S8:	$\Sigma [\sin(2\pi f_r)/C_n][\cos(2\pi f_r)/C_n]$
R9:		S9:	n
RA:	P_n or $P(1)$		
RB:	$(2\pi f_r)/C_n$ and $\cos(2\pi f_r)/C_n$		
RC:	$\sin(2\pi f_r)/C_n$		
RD:	Coherent TL		
RE:	Incoherent TL		
RI:	Scratch		

Initial Flag Status and Use: OFF, Unused

Trig Mode: RAD

User Control Keys:

A: $f \uparrow z_0 \uparrow z \uparrow P(1)$ or $-|P_n|$ a:

B: $\alpha_n \uparrow r \uparrow C_n \rightarrow P_n$ or $P(1)$ b:

C: Sum sin, $\cos(2\pi f_r)/C_n$ c:

D: Sum P_n^2 d:

E: $\rightarrow$ Coherent TL e: $\rightarrow$ Incoherent TL

001	*LBLA	21 11	Enter values for f, Z_0, Z, and either P(1) or $-\lvert P_n \rvert$. Clears all registers, sets radian mode.
002	RAD	16-22	
003	CLRG	16-53	
004	P⇄S	16-51	
005	CLRG	16-53	
006	STO6	35 06	
007	R↓	-31	
008	STO2	35 02	
009	R↓	-31	
010	STO1	35 01	
011	R↓	-31	
012	STO0	35 00	
013	RTN	24	
014	*LBLB	21 12	Enter values for α_n, r, and c_n
015	STO3	35 03	
016	R↓	-31	
017	STO5	35 05	Determine calculations required. 1 is stored in R_9 if P_n is being calculated
018	R↓	-31	
019	STO4	35 04	
020	RCL6	36 06	
021	X<0?	16-45	
022	GTO1	22 01	
023	1	01	
024	STO9	35 09	
025	*LBL2	21 02	Set up storage for Z_n (Z_0) and Z_n (z); branches to user-defined function to compute
026	7	07	
027	STOI	35 46	
028	RCL1	36 01	
029	GSB3	23 03	
030	8	08	
031	STOI	35 46	
032	RCL2	36 02	
033	GSB3	23 03	
034	RCL9	36 09	
035	X<0?	16-45	Returns to Label 1 for calculations of P(1)
036	RTN	24	
037	*LBL4	21 04	Compute P_n. *Note that $\lvert P_n \rvert$ is stored in Register A, $\lvert P_n \rvert$ is displayed upon completion.
038	RCL7	36 07	
039	RCL5	36 05	
040	√X	54	
041	÷	-24	
042	RCL8	36 08	
043	×	-35	
044	RCL3	36 03	
045	RCL0	36 00	
046	÷	-24	
047	√X	54	
048	×	-35	
049	RCL6	36 06	
050	×	-35	
051	RCL4	36 04	
052	RCL5	36 05	
053	×	-35	
054	CHS	-22	
055	e^x	33	
056	×	-35	
057	STOA	35 11	

058	SCI	-12	
059	ABS	16 31	
060	RTN	24	
061	*LBL1	21 01	Stores in $\lvert P_n \rvert$ in R_6. -1 stores in R_9 to indicate that P(1) is being calculated.
062	CHS	-22	
063	STO6	35 06	
064	1	01	
065	CHS	-22	
066	STO9	35 09	
067	GSB2	23 02	Calculate $Z_n(z_0)$, $Z_n(z)$
068	RCL5	36 05	Compute P(1)
069	√X	54	
070	RCL6	36 06	
071	x	-35	
072	RCL3	36 03	
073	RCL0	36 00	
074	÷	-24	
075	√X	54	
076	÷	-24	
077	RCL7	36 07	
078	÷	-24	
079	RCL8	36 08	
080	÷	-24	
081	RCL4	36 04	
082	RCL5	36 05	
083	x	-35	
084	CHS	-22	
085	e^x	33	
086	÷	-24	
087	X<0?	16-45	
088	CHS	-22	
089	STOA	35 11	
090	SCI	-12	
091	RTN	24	
092	*LBL3	21 03	User-defined label to compute $Z_n(z_0)$ and $Z_n(z)$.
093	STOi	35 45	
094	RTN	24	
095	*LBLC	21 13	Sums and stores the sine and cosine of $\frac{2\pi f_r}{C_n}$ for use in calculations of T.L. (coherent)
096	RCL5	36 05	
097	RCL0	36 00	
098	x	-35	
099	Pi	16-24	
100	2	02	
101	x	-35	
102	x	-35	
103	RCL3	36 03	
104	÷	-24	
105	STOB	35 12	
106	SIN	41	
107	STOC	35 13	
108	RCLB	36 12	
109	COS	42	
110	STOB	35 12	
111	RCLA	36 11	
112	x	-35	
113	RCLC	36 13	
114	RCLA	36 11	

115	×	-35	
116	Σ+	56	
117	RTN	24	
118	*LBLD	21 14	
119	RCLA	36 11	
120	X²	53	
121	P⇄S	16-51	
122	ST+0	35-55 00	Computes P_n^2
123	P⇄S	16-51	
124	RTN	24	
125	*LBLE	21 15	
126	P⇄S	16-51	
127	RCL4	36 04	
128	X²	53	
129	RCL6	36 06	
130	X²	53	
131	+	-55	
132	√X	54	
133	P⇄S	16-51	
134	RCL6	36 06	Computes T.L.
135	÷	-24	assuming coherence.
136	LOG	16 32	
137	2	02	
138	0	00	
139	×	-35	
140	CHS	-22	
141	FIX	-11	
142	DSP0	-63 00	
143	STOD	35 14	
144	RTN	24	
145	*LBLe	21 16 15	
146	P⇄S	16-51	
147	RCL0	36 00	
148	P⇄S	16-51	
149	√X	54	
150	RCL6	36 06	
151	÷	-24	
152	LOG	16 32	Computes T.L.
153	2	02	assuming incoherence.
154	0	00	
155	×	-35	
156	CHS	-22	
157	FIX	-11	
158	DSP0	-63 00	
159	STOE	35 15	
160	RTN	24	
161	R/S	51	

E. Mathematical Analysis

The following four formulas are the basis for the mathematical computations (Ref. 1).

1. $$P_n = P(1) \sqrt{\frac{C_n}{f}} \; \frac{Z_n(z_0)}{\sqrt{r}} \; Z_n(z) \; \exp(-\alpha_n r)$$

2. $$P(1) = \frac{P_n \sqrt{r}}{\sqrt{(C_n/f)} \; Z_n(z_0) \; Z_n(z) \; \exp(-\alpha_n r)}$$

3. Coherent transmission loss

$$TL = -20 \log \left[\left[\left(\Sigma P_n \sin \frac{2f_r}{C_n} \right)^2 + \left(\Sigma P_n \cos \frac{2f_r}{C_n} \right)^2 \right]^{1/2} \Big/ P(1) \right]$$

4. Incoherent transmission loss

$$TL = -20 \log \left[(\Sigma P_n^2)^{1/2} / P(1) \right]$$

F. Reference

1. A. B. Coppens and J. V. Sanders, "An Introduction to the Sonar Equations with Applications," Technical Report NPS-61Sd76071, July 1976, Naval Postgraduate School, Monterey, CA 93940

XI. GOLDEN SECTION SEARCH by LT J. K. McDermott

A. Problem Statement

The minimum value of a unimodal function of one variable, f(x), for a specified interval is determined by utilizing Golden Section Search Techniques, i.e.

$$\text{Minimize} \quad f(x)$$
$$\text{Subject to} \quad x \in I$$

where $I = [a,b]$ is a closed interval in E_1 space.

B. Operational Analysis

Golden Section Search is a specific type of interval of uncertainty (IOU) method of single variable optimization which requires the selection of a specific interval. Once the interval has been selected, the program locates the value of x which will minimize a unimodal function f(x) being evaluated within this specific interval. If a different interval is selected, a different x with a correspondingly different minimum functional value may be obtained.

Golden Section Search locates a local minimum and not the global minimum. A function which is not unimodal over the specific interval may produce an x value which does not provide the minimum (global) functional value within the interval. The behavior for non-unimodal functions is not predictable.

The functions which may be evaluated are limited to some degree by (i) the number of program steps available for user supplied function program listing (139-224), (ii) functions of one variable preferably unimodal over the IOU to avoid ambiguity, and (iii) user's programming capability and imagination.

Golden Section Search was originally suggested by J. Kiefer, "Sequential Minimax Search for a Maximum," Proc. Amer. Math. Soc., 4, no. 3, June 1953, pp. 502-506. The name traces back to Euclid's discovery that it is possible to divide any given line segment into two parts such that the ratio of the whole to the larger part equals the ratio of the larger part to the smaller. The division of a line in this manner came to be known as the Golden Section, both because it has several rather interesting geometric and numerical properties and because the proportions of the two parts seem pleasing to the eye.

The author (programmer) is indebted to Professor J. K. Hartman of the Naval Postgraduate School whose lectures and class notes form the bases of this HP-67/97 calculator program.

C. Computational Algorithm

Basic IOU Algorithm Structure (GSS)

1. Given initial IOU I = [a,b] and function f(x). Let K be a function evaluation, iteration counter.

2. Compute initial X_1 as

$$X_1 = a + \sigma(b-a) \quad \text{with} \quad \sigma = (3 - \sqrt{5})/2$$

Set $I_1 = 1$ and $K = 2$.

3. At iteration K, interval I_{K-1} resulting from previous iteration contains best point (one producing smaller function value) thus far and its relative position is σ or $1-\sigma$. Place new point (X_K) symmetrically:

$$X_K = \text{ENDL} + \text{ENDR} - X_{\text{OLD}}$$

where $I_{K-1} = [\text{ENDL}, \text{ENDR}]$ and X_{OLD} provided the smaller function value between previous two evaluated points.

4. Compute $f(X_K)$.

5. Shorten IOU to $I_K \subset I_{K-1}$ with length $L_K < L_{K-1}$ from information $f(X_K)$ provides. Set $K = K+1$ and go to step 3 for the next iteration.

STOPPING RULE:

Stop when either K = NMAX (present number of function iterations) or when $L_K \leq$ RIOU (preset required interval of uncertainty length).

HP-67 Computational Algorithm

1. Input user supplied function program listing in available program steps 139 through 224.
2. Input left endpoint of interval of uncertainty (ENDL).
3. Input right endpoint of interval of uncertainty (ENDR).
4. Input required length of the final interval of uncertainty (RIOU).
5. Input maximum number of function evaluations desired (NMAX)
6. Output final interval of uncertainty [ENDL,ENDR].
7. Output minimum (local) function value in interval.
8. Output X value that produces minimum function value.

D. HP-67/97 Calculator Program

1. User Instructions

Step	Instruction	Input	Key(s)	Output
1.	Enter program card			
2.	Select GTO f e		GTO f e	
3.	Slide W/PRGM-RUN switch to W/PRGM	User supplied function program		
4.	Slide W/PRGM-RUN switch to RUN			
5.	Enter left endpoint of interval of uncertainty	ENDL	A	ENDL
6.	Enter right endpoint of interval of uncertainty	ENDL	B	ENDR
7.	Enter required length of interval of uncertainty	RIOU	C	RIOU
8.	Enter maximum number of function evaluations to be performed	NMAX	D	NMAX
9.	Compute final interval of uncertainty, minimum function value, and corresponding X value	NONE	E	
	ENDL displayed when computation complete			ENDL
10.	Press R/S to display ENDR			ENDR
11.	Press R/S again to display f(X)			f(X)
12.	Press R/S once more to display X			X
13.	To repeat program press f CLREG and go to Step 5		f CL REG	

2. Sample Problem

a. minimize $f(x) = |x^2 - 16|$

subject to $x \in I_0$

$I_0 = [1,16]$, RIOU = 0.01, NMAX = 25.

SOLUTION: [3.99560, 4.00240] in 17 function evaluations

Function Value = 0.00162

Minimum Point x = 3.99980

User Supplied Program Listing

GTOe 139 *LBLe 21 16 15 140 RCLE 36 15 141 X² 53 142 1 01 143 6 06 144 - -45 145 ABS 16 31 146 RTN 24 147 R/S 51 1.00 GSBA 16.00 GSBB .61 GSBC 25.00 GSBD GSBE 3.99560 *** R/S 4.00240 *** R/S 0.00162 *** R/S 3.99980 ***	Recalls x from Register E. Function value is left in the display (x-register).

NOTE: First load provided program (with RUN position). Select GTO f e. Move switch to W/PRGM. Enter user supplied program for function. Move switch to run position. Enter input and compute.

b. Minimize $f(x) = 20 - x + \frac{1}{(16-x)}$

Subject to $x \in I_0$

$I_0 = [10, 15.9]$, RIOU = 0.01, NMAX = 25

SOLUTION: [14.99555, 15.00255] in 15 function evaluations

Function Value = 6.00000

Minimum Point x = 14.99823

User Supplied Program Listing

GTOe 139 *LBLe 21 16 15 140 2 02 141 0 00 142 RCLE 36 15 143 - -45 144 1 01 145 6 06 146 RCLE 36 15 147 - -45 148 1/X 52 149 + -55 150 RTN 24 151 R/S 51 10.00 GSBA 15.90 GSBB .01 GSBC 25.00 GSBD GSBE 14.99555 *** R/S 15.00255 *** R/S 6.00000 *** R/S 14.99623 ***	Recall x from Register E. Result is left in x-register

NOTE: First load provided program with switch in RUN position. Select GTO f e. Move switch to W/PRGM. Enter user supplied program for function. Move switch to RUN position. Enter input and compute.

c. minimize $f(x) = \frac{x}{2} + \sin(\frac{\pi x}{2})$ (x in radians)

subject to $x \in I_0$

$I_0 = [0,10]$, RIOU = 0.01, NMAX = 25.

SOLUTION: [2.79094, 2.79827] in 16 function evaluations

$$\left.\begin{array}{l}\text{Function value} = 0.44890\\ \text{Minimum point } x = 2.79373\end{array}\right\}\begin{array}{l}\text{local minimum}\\ (x = 0.0 \text{ is global})\end{array}$$

User Supplied Program Listing

<pre>   GTOe
139 *LBLe 21 16 15
140   RCLE   36 15
141   2   02
142   ÷   -24
143   ENT↑   -21
144   ENT↑   -21
145   Pi   16-24
146   x   -35
147   SIN   41
148   +   -55
149   RTN   24
150   R/S   51
 	
          0.00 GSBa
         10.00 GSBb
           .01 GSBc
         25.00 GSBd
               GSBE
       2.79094 ***
                R/S
       2.79827 ***
                R/S
       0.44890 ***
                R/S
       2.79373 ***</pre>
	Recalls R5 for x utilization and
	
	stores function value in R7
	
	(GSB 1)

NOTE: First load provided program with switch in RUN position Select GTO f e. Move switch to W/PRGM. Enter user supplied program for function. Move switch to RUN position. Enter input and compute.

3. Program Storage Allocation

Registers:

R0: function counter	S0:	RA:
R1: ENDL	S1:	RB:
R2: ENDR	S2:	BC:
R3: RIOU	S3:	RD:
R4: NMAX	S4:	RE: Current X
R5: X_1	S5:	RI: 4 (decrement)
R6: X_2	S6:	
R7: F_1	S7:	
R8: F_2	S8:	
R9: not used	S9:	

NOTE: User supplied function can utilize sixteen registers.

Initial Flag Status and Use:

0: OFF, Unused	2: OFF, Unused
1: OFF, Unused	3: OFF, Unused

User Control Keys

A: Left endpoint (ENDL)	a:
B: Right endpoint (ENDR)	b:
C: Required interval of uncertainty (RIOU)	c:
D: Maximum function iterations (NMAX)	d:
E: Compute	e: User defined function

*LBLA	21 11	Input: Left endpoint Right endpoint Required interval of uncertainty Maximum number of function evaluations (ENDL, ENDR, RIOU, NMAX)
STO1	35 01	
RTN	24	
*LBLB	21 12	
STO2	35 02	
RTN	24	
*LBLC	21 13	
STO3	35 03	
RTN	24	
*LBLD	21 14	
STO4	35 04	
3	03	Calculate Sigma $\sigma = \frac{(3 - \sqrt{5})}{2}$
ENT↑	-21	
5	05	
√X	54	
-	-45	
2	02	
÷	-24	
RCL2	36 02	Calculate initial X_1 X_1 = ENDL + σ(ENDR - ENDL)
RCL1	36 01	
-	-45	
×	-35	
RCL1	36 01	
+	-55	
STO5	35 05	
0	00	Initialize counters Set up display Require radian calculations
STO6	35 06	
4	04	
STOI	35 46	
RCL4	36 04	
DSP5	-63 05	
RAD	16-22	
RTN	24	
*LBLE	21 15	Calculate initial X_2
RCL5	36 05	
RCL1	36 01	
-	-45	
RCL2	36 02	
X⇄Y	-41	
-	-45	
STO6	35 06	
GSB1	23 01	Obtain initial function values (F_1 and F_2) from initial X_1 and X_2.
GSB8	23 08	
GSB2	23 02	
GSB8	23 08	

046	*LBLa	21 16 11	Determine larger function value. If F_1 larger go to branch two, otherwise go to branch one.
047	RCL8	36 08	
048	RCL7	36 07	
049	X>Y?	16-34	
050	GTO3	22 03	
051	RCL6	36 06	X_2 becomes right endpoint of interval of uncertainty
052	STO2	35 02	
053	RCL1	36 01	Compare IOU < RIOU? If yes, print out final results
054	-	-45	
055	RCL3	36 03	
056	X⇄Y	-41	
057	X≤Y?	16-35	
058	GTO4	22 04	
059	RCL5	36 05	Old X_1 becomes X_2 Old F_1 becomes F_2
060	STO6	35 06	
061	RCL7	36 07	
062	STO8	35 08	
063	GSB6	23 06	Determines new $X \Rightarrow (X_1)$ and store.
064	STO5	35 05	
065	RCL4	36 04	Has NMAX been exceeded? If yes, print error.
066	RCL0	36 00	
067	X>Y?	16-34	
068	GTO0	22 00	
069	GSB1	23 01	Calculate F_1 and return to compare new values of F_1 and F_2.
070	GSB8	23 08	
071	GTOa	22 16 11	
072	*LBL3	21 03	X_1 becomes left endpoint of interval of uncertainty
073	RCL2	36 02	
074	RCL5	36 05	
075	STO1	35 01	
076	-	-45	Compare IOU < RIOU? If yes, print out final results
077	RCL3	36 03	
078	X⇄Y	-41	
079	X≤Y?	16-35	
080	GTO4	22 04	
081	RCL6	36 06	Old X_2 becomes X_1 Old F_2 becomes F_1
082	STO5	35 05	
083	RCL8	36 08	
084	STO7	35 07	
085	GSB6	23 06	Determines new $X \Rightarrow (X_2)$ and store.
086	STO6	35 06	
087	RCL4	36 04	Has NMAX been exceeded? If yes, print error.
088	RCL0	36 00	
089	X>Y?	16-34	
090	GTO0	22 00	

091	GSB2	23 02	Calculate F_2 and return to compare new values of F_1 and F_2
092	GSB8	23 08	
093	GTOa	22 16 11	
094	*LBL8	21 08	Increments function counter for determination of NMAX exceeded.
095	1	01	
096	ST+0	35-55 00	
097	RTN	24	
098	*LBL6	21 06	Determines new X (X_1 or X_2 depending on which branch subroutine called from)
099	RCL2	36 02	
100	RCL5	36 05	
101	-	-45	
102	RCL1	36 01	
103	+	-55	
104	RTN	24	
105	*LBL4	21 04	Determines which function value with corresponding ENDL, ENDR, and X should be printed. (Smaller function value used.) used.)
106	RCL8	36 08	
107	RCL7	36 07	
108	X>Y?	16-34	
109	GTO5	22 05	
110	RCL5	36 05	Sets up stack for printout if F_1 small value.
111	RCL7	36 07	
112	RCL2	36 02	
113	RCL1	36 01	
114	GTO7	22 07	
115	*LBL5	21 05	Sets up stack for printout if F_2 smaller value
116	RCL6	36 06	
117	RCL8	36 08	
118	RCL2	36 02	
119	RCL1	36 01	
120	*LBL7	21 07	Prints out final results.
121	R/S	51	
122	R↓	-31	
123	DSZI	16 25 46	
124	GTO7	22 07	
125	GTO0	22 00	
126	RTN	24	
127	*LBL1	21 01	F_1 Routine
128	RCL5	36 05	
129	STOE	35 15	
130	GSBe	23 16 15	
131	STO7	35 07	
132	RTN	24	
133	*LBL2	21 02	F_2 Routine
134	RCL6	36 06	
135	STOE	35 15	
136	GSBe	23 16 15	
137	STO8	35 08	
138	RTN	24	
139	*LBLe	21 16 15	Use defined function
140	RTN	24	
141	R/S	51	

E. Mathematical Analysis

For each X the function $f(X)$ is evaluated. By comparing two function values F_1 and F_2 the interval of uncertainty can be reduced as follows:

The placement of the X's is determined by the Golden Section Search Technique utilizing the "Golden Ratio." Placement of the first X (X_1) determines the placement of all other X's since all remaining points are placed symmetrically with respect to each point remaining in successive intervals of uncertainty.

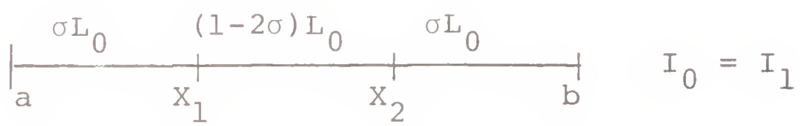
In order to ensure that each IOU length is predictively independent of the function $f(x)$, X_1 and X_2 are placed symmetrically in the IOU. When the IOU is reduced, the new shorter IOU still contains the best point thus far achieved, so selecting a new X point will allow further reduction. (New X for each iteration.)

Golden Section Search selects X_1 to satisfy:

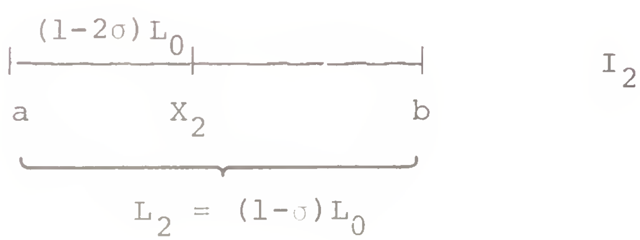
The <u>relative position</u> of the X points in the remaining IOU is the same at each iteration.

Explanation:

$I_1 = I_0$ since no reduction with one point. Relative position of X_1 in I_1 is $GL_0/L_0 = \sigma$. Now place X_2 <u>symmetrically</u>



Suppose $f(X_2) < f(X_1)$ and reduce IOU accordingly



Relative position of X_2 is

$$\frac{(1 - 2\sigma)L_0}{(1 - \sigma)\ L_0} = \frac{1 - 2\sigma}{1 - \sigma}$$

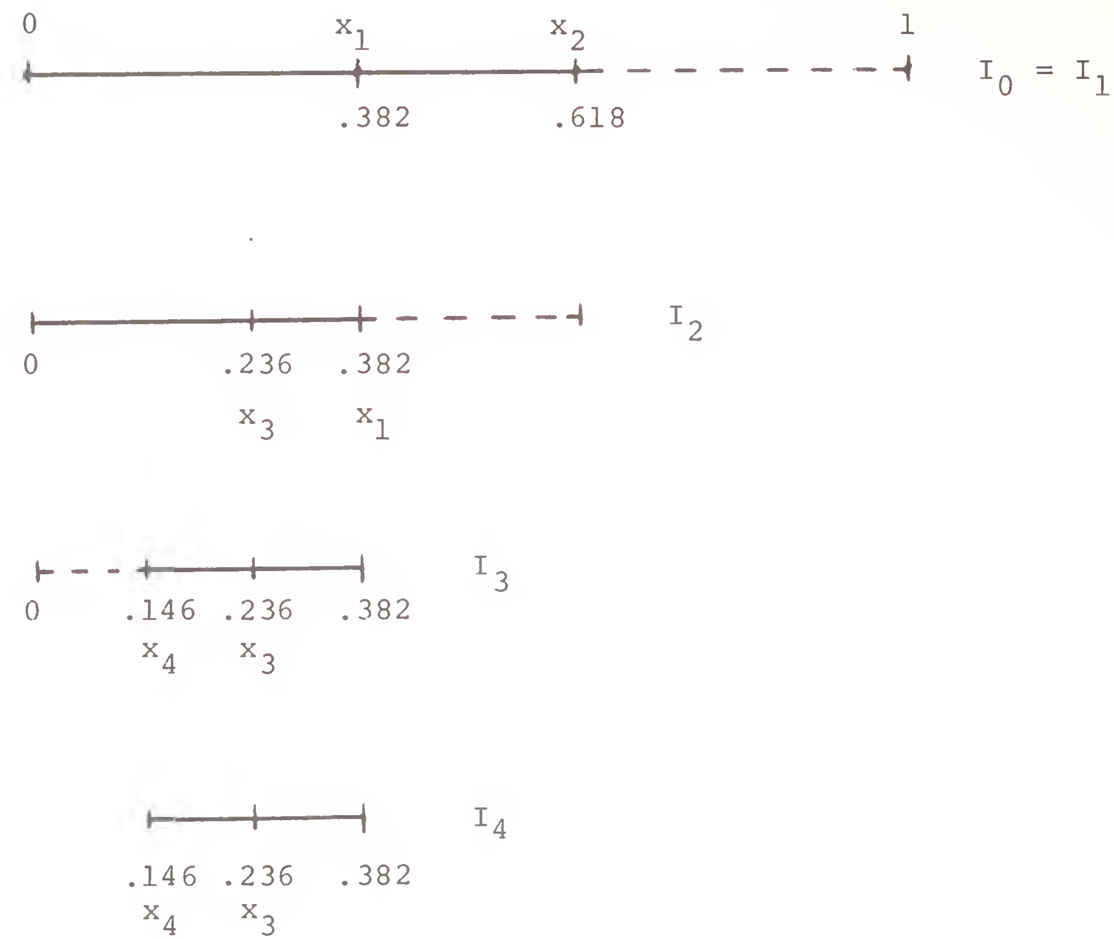
$\sigma = \dfrac{1 - 2\sigma}{1 - \sigma}$ ==> ("relative position same at each iteration")

$\sigma^2 - 3\sigma + 1 = 0 \implies \sigma = \dfrac{3 \pm \sqrt{5}}{2}$ use root $\sigma = \dfrac{3 - \sqrt{5}}{2}$

Choose $X_1 = \sigma b + (1-\sigma)a = a + \sigma(b-a)$.

<u>Example</u>: $n = 4$, $f(x) = (X - .303)^2$, $I_0 = [0,1]$.

K	I_{K-1}	X_K	$f(X_K)$	I_K	L_K
0	--	--	--	[0,1]	1
1	[0,1]	.382	$(.079)^2$	[0,1]	1
2	[0,1]	.618	$(.315)^2$	[0,.618]	$.618 = (1-\sigma)$
3	[0,.618]	.236	$(.067)^2$	[0,.382]	$.382 = (1-\sigma)^2$
4	[0,.382]	.146	$(.157)^2$	[.146,.382]	$.236 = (1-\sigma)^3$



FINAL ANSWER: $X_.$ (OPTIMAL) e[.146,.382];

X_3 = .236 is best point evaluated thus far.

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